Introduction

1. Geometric figures. The part of space occupied by a physical object is called a **geometric solid**.

A geometric solid is separated from the surrounding space by a **surface**.

A part of the surface is separated from an adjacent part by a **line**.

A part of the line is separated from an adjacent part by a **point**.

The geometric solid, surface, line and point do not exist separately. However by way of abstraction we can consider a surface independently of the geometric solid, a line — independently of the surface, and the point — independently of the line. In doing so we should think of a surface as having no thickness, a line — as having neither thickness nor width, and a point — as having no length, no width, and no thickness.

A set of points, lines, surfaces, or solids positioned in a certain way in space is generally called a **geometric figure**. Geometric figures can move through space without change. Two geometric figures are called **congruent**, if by moving one of the figures it is possible to superimpose it onto the other so that the two figures become identified with each other in all their parts.

2. Geometry. A theory studying properties of geometric figures is called **geometry**, which translates from Greek as *land-measuring.* This name was given to the theory because the main purpose of geometry in antiquity was to measure distances and areas on the Earth's surface.

First concepts of geometry as well as their basic properties, are introduced as idealizations of the corresponding common notions and everyday experiences.

3. The plane. The most familiar of all surfaces is the flat surface, or the **plane**. The idea of the plane is conveyed by a window pane, or the water surface in a quiet pond.

We note the following property of the plane: *One can superimpose a plane on itself or any other plane in a way that takes one given point to any other given point, and this can also be done after flipping the plane upside down.*

4. The straight line. The most simple line is the **straight line**. The image of a thin thread stretched tight or a ray of light emitted through a small hole give an idea of what a straight line is. The following fundamental property of the straight line agrees well with these images:

For every two points in space, there is a straight line passing through them, and such a line is unique.

It follows from this property that:

If two straight lines are aligned with each other in such a way that two points of one line coincide with two points of the other, then the lines coincide in all their other points as well (because otherwise we would have two distinct straight lines passing through the same two points, which is impossible).

For the same reason, *two straight lines can intersect at most at one point.*

A straight line can lie in a plane. The following holds true:

If a straight line passes through two points of a plane, then all points of this line lie in this plane.

Figure 1 Figure 2 Figure 3

5. The unbounded straight line. Ray. Segment. Thinking of a straight line as extended indefinitely in both directions, one calls it an **infinite** (or **unbounded**) straight line.

A straight line is usually denoted by two uppercase letters marking any two points on it. One says "the line AB" or "BA" (Figure 1).

A part of the straight line bounded on both sides is called a **straight segment.** It is usually denoted by two letters marking its endpoints (the segment CD, Figure 2). Sometimes a straight line or a segment is denoted by one (lowercase) letter; one may say "the straight line a, the segment b."

Usually instead of "unbounded straight line" and "straight segment" we will simply say **line** and **segment** respectively.

Sometimes a straight line is considered which terminates in one direction only, for instance at the endpoint E (Figure 3). Such a straight line is called a **ray** (or **half-line**) drawn from E.

6. Congruent and non-congruent segments. *Two segments are congruent if they can be laid one onto the other so that their endpoints coincide.* Suppose for example that we put the segment AB onto the segment CD (Figure 4) by placing the point A at the point C and aligning the ray AB with the ray CD. If, as a result of this, the points B and D merge, then the segments AB and CD are congruent. Otherwise they are not congruent, and the one which makes a part of the other is considered smaller.

Figure 4

To mark on a line a segment congruent to a given segment, one uses the **compass**, a drafting device which we assume familiar to the reader.

7. Sum of segments. The sum of several given segments (AB, CD, EF, Figure 5) is a segment which is obtained as follows. On a line, pick any point M and starting from it mark a segment MN congruent to AB, then mark the segments NP congruent to CD, and PQ congruent to EF, both going in the same direction as MN. Then the segment MQ will be the sum of the segments AB, CD and EF (which are called **summands** of this sum). One can similarly obtain the sum of any number of segments.

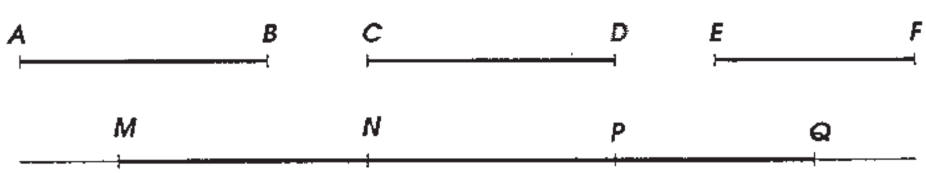

Figure 5

The sum of segments has the same properties as the sum of numbers. In particular it does not depend on the order of the summands (the **commutativity** law) and remains unchanged when some of the summands are replaced with their sum (the **associativity** law). For instance:

$$AB + CD + EF = AB + EF + CD = EF + CD + AB = \dots$$

and

$$AB + CD + EF = AB + (CD + EF) = CD + (AB + EF) = \dots.$$

8. Operations with segments. The concept of addition of segments gives rise to the concept of subtraction of segments, and multiplication and division of segments by a whole number. For example, the difference of AB and CD (if $AB > CD$) is a segment whose sum with CD is congruent to AB; the product of the segment AB with the number 3 is the sum of three segments each congruent to AB; the quotient of the segment AB by the number 3 is a third part of AB.

If given segments are measured by certain linear units (for instance, centimeters), and their lengths are expressed by the corresponding numbers, then the length of the sum of the segments is expressed by the sum of the numbers measuring these segments, the length of the difference is expressed by the difference of the numbers, etc.

9. The circle. If, setting the compass to an arbitrary step and, placing its pin leg at some point O of the plane (Figure 6), we begin to turn the compass around this point, then the other leg equipped with a pencil touching the plane will describe on the plane a continuous curved line all of whose points are the same distance away from O. This curved line is called a **circle**, and the point O — its **center**. A segment (OA, OB, OC in Figure 6) connecting the center with a point of the circle is called a **radius**. All radii of the same circle are congruent to each other.

Circles described by the compass set to the same radius are congruent because by placing their centers at the same point one will identify such circles with each other at all their points.

A line (MN, Figure 6) intersecting the circle at any two points is called a **secant**.

A segment (EF) both of whose endpoints lie on the circle is called a **chord**.

A chord (AD) passing through the center is called a **diameter**. A diameter is the sum of two radii, and therefore all diameters of the same circle are congruent to each other.

A part of a circle contained between any two points (for example, EmF) is called an **arc**.

The chord connecting the endpoints of an arc is said to **subtend** this arc.

An arc is sometimes denoted by the sign $\frown$; for instance, one writes: $\overset{\frown}{EmF}$.

The part of the plane bounded by a circle is called a **disk.**[2]

The part of a disk contained between two radii (the shaded part COB in Figure 6) is called a **sector**, and the part of the disk cut off by a secant (the part EmF) is called a **disk segment**.

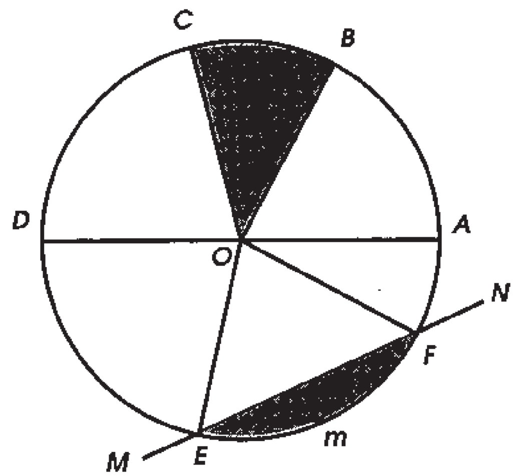

Figure 6

10. Congruent and non-congruent arcs. *Two arcs of the same circle (or of two congruent circles) are congruent if they can be aligned so that their endpoints coincide.* Indeed, suppose that we align the arc AB (Figure 7) with the arc CD by identifying the point A with the point C and directing the arc AB along the arc CD. If, as a result of this, the endpoints B and D coincide, then all the intermediate points of these arcs will coincide as well, since they are the same distance away from the center, and therefore $\overset{\frown}{AB}=\overset{\frown}{CD}$. But if B and D do not coincide, then the arcs are not congruent, and the one which is a part of the other is considered smaller.

11. Sum of arcs. The sum of several given arcs of the same radius is defined as an arc of that same radius which is composed from parts congruent respectively to the given arcs. Thus, pick an arbitrary point M (Figure 7) of the circle and mark the part MN

[2]Often the word "circle" is used instead of "disk." However one should avoid doing this since the use of the same term for different concepts may lead to mistakes.

congruent to AB. Next, moving in the same direction along the circle, mark the part NP congruent to CD. Then the arc MP will be the sum of the arcs AB and CD.

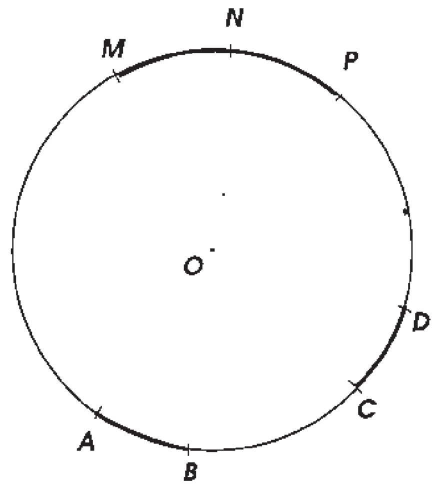

Figure 7

Adding arcs of the same radius one may encounter the situation when the sum of the arcs does not fit in the circle and one of the arcs partially covers another. In this case the sum will be an arc greater than the whole circle. For example, adding the arcs AmB and CnD (Figure 8) we obtain the arc consisting of the whole circle and the arc AD.

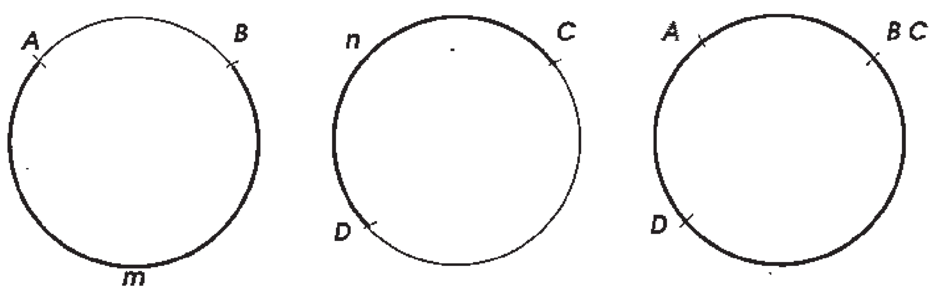

Figure 8

Similarly to addition of line segments, addition of arcs obeys the commutativity and associativity laws.

From the concept of addition of arcs one derives the concepts of subtraction of arcs, and multiplication and division of arcs by a whole number the same way as it was done for line segments.

12. Divisions of geometry. The subject of geometry can be divided into two parts: **plane geometry**, or **planimetry**, and **solid geometry**, or **stereometry**. Planimetry studies properties of those geometric figures all of whose elements fit the same plane.