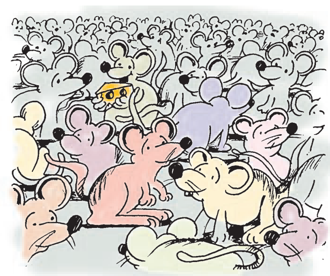
B EXPONENTIAL FUNCTIONS [3.2, 3.3, 3.5, 3.8]

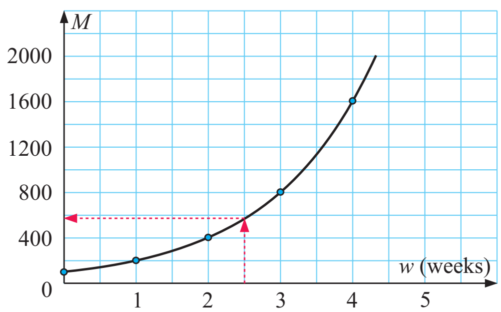


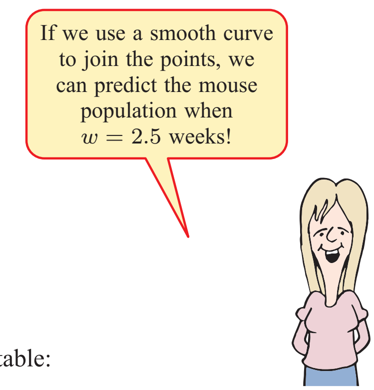
Consider a population of 100 mice which is growing under plague conditions.

If the mouse population doubles each week, we can construct a **table** to show the population number M after w weeks.

w (weeks)	0	1	2	3	4	......
M	100	200	400	800	1600	......

We can also represent this information on a graph as:





We can find a relationship between M and w using another table:

w	M values
0	$100 = 100 \times 2^0$
1	$200 = 100 \times 2^1$
2	$400 = 100 \times 2^2$
3	$800 = 100 \times 2^3$
4	$1600 = 100 \times 2^4$

So, we can write $M = 100 \times 2^w$.

This is an **exponential function** and the graph is an **exponential graph**.

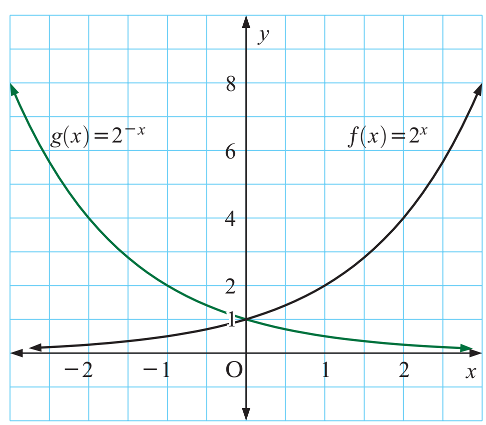
We can use the function to find M for any value of $w \geqslant 0$.

For example, when $w = 2.5$, $M = 100 \times 2^{2.5}$
≈ 566 mice

An **exponential function** is a function in which the variable occurs as part of the exponent or index.

The simplest exponential functions have the form $f(x) = a^x$ where a is a positive constant, $a \neq 1$.

For example, graphs of the exponential functions $f(x) = 2^x$ and $g(x) = (\frac{1}{2})^x = 2^{-x}$ are shown alongside.



Discovery 2 — Graphs of simple exponential functions

GRAPHING PACKAGE

This discovery is best done using a graphing package or graphics calculator.

What to do:

1 **a** On the same axes, graph $y = (1.2)^x$, $y = (1.5)^x$, $y = 2^x$, $y = 3^x$, $y = 7^x$.
b State the coordinates of the point which all of these graphs pass through.
c Explain why $y = a^x$ passes through this point for all $a \in \mathbb{R}$, $a > 0$.
d State the equation of the asymptote common to all these graphs.
e Comment on the shape of the family of curves $y = a^x$ as a increases in value.

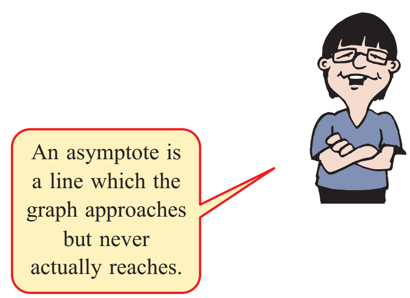
2 On the same set of axes graph $y = (\frac{1}{3})^x$ and $y = 3^{-x}$.
Explain your result.

3 **a** On the same set of axes graph $y = 3 \times 2^x$, $y = 6 \times 2^x$ and $y = \frac{1}{2} \times 2^x$.
b State the y-intercept of $y = k \times 2^x$. Explain your answer.

4 **a** On the same set of axes graph $y = 5 \times 2^x$ and $y = 5 \times 2^{-x}$.
b What is the significance of the factor 5 in each case?
c What is the difference in the shape of these curves, and what causes it?

All graphs of the form $f(x) = a^x$ where a is a positive constant not equal to 1:

- have a **horizontal asymptote** $y = 0$ (the x-axis)
- pass through (0, 1) since $f(0) = a^0 = 1$.



Example 3 — Self Tutor

For the function $f(x) = 3 - 2^{-x}$, find: **a** $f(0)$ **b** $f(3)$ **c** $f(-2)$

a $f(0) = 3 - 2^0$
$= 3 - 1$
$= 2$

b $f(3) = 3 - 2^{-3}$
$= 3 - \frac{1}{8}$
$= 2\frac{7}{8}$

c $f(-2) = 3 - 2^{-(-2)}$
$= 3 - 2^2$
$= 3 - 4$
$= -1$

EXERCISE 28B

1 If $f(x) = 3^x + 2$, find the value of: **a** $f(0)$ **b** $f(2)$ **c** $f(-1)$

2 If $f(x) = 5^{-x} - 3$, find the value of: **a** $f(0)$ **b** $f(1)$ **c** $f(-2)$

3 If $g(x) = 3^{x-2}$, find the value of: **a** $g(0)$ **b** $g(4)$ **c** $g(-1)$