Example 4 — Self Tutor

Find, using your calculator: **a** 6^5 **b** $(-5)^4$ **c** -7^4

		Answer
a	Press: 6 [^] 5 [ENTER]	7776
b	Press: [( ] [(−)] 5 [ )] [^] 4 [ENTER]	625
c	Press: [(−)] 7 [^] 4 [ENTER]	−2401

EXERCISE 6A.2

1 Simplify:

a $(-1)^4$ **b** $(-1)^5$ **c** $(-1)^{10}$ **d** $(-1)^{15}$ **e** $(-1)^8$ **f** -1^8

g $-(-1)^8$ **h** $(-3)^3$ **i** -3^3 **j** $-(-3)^3$ **k** $-(-6)^2$ **l** $-(-4)^3$

2 Simplify:

a $2^3 \times 3^2 \times (-1)^5$ **b** $(-1)^4 \times 3^3 \times 2^2$ **c** $(-2)^3 \times (-3)^4$

3 Use your calculator to find the value of the following, recording the entire display:

a 2^8 **b** $(-5)^4$ **c** -3^5 **d** 7^4 **e** 8^3 **f** $(-7)^6$

g -7^6 **h** 1.05^{12} **i** -0.623^{11} **j** $(-2.11)^{17}$

B EXPONENT OR INDEX LAWS [1.9, 2.4]

Notice that:

- $2^3 \times 2^4 = 2 \times 2 \times 2 \ \times \ 2 \times 2 \times 2 \times 2 = 2^7$
- $\dfrac{2^5}{2^2} = \dfrac{2 \times 2 \times 2 \times \cancel{2 \times 2}^{\,1}}{\cancel{2 \times 2}_{\,1}} = 2^3$
- $(2^3)^2 = 2 \times 2 \times 2 \ \times \ 2 \times 2 \times 2 = 2^6$
- $(3 \times 5)^2 = 3 \times 5 \times 3 \times 5 = 3 \times 3 \times 5 \times 5 = 3^2 5^2$
- $\left(\dfrac{2}{5}\right)^3 = \dfrac{2}{5} \times \dfrac{2}{5} \times \dfrac{2}{5} = \dfrac{2 \times 2 \times 2}{5 \times 5 \times 5} = \dfrac{2^3}{5^3}$

These examples can be generalised to the exponent or index laws:

- $a^m \times a^n = a^{m+n}$ — To **multiply** numbers with the **same base**, keep the base and **add** the indices.
- $\dfrac{a^m}{a^n} = a^{m-n}, \quad a \neq 0$ — To **divide** numbers with the **same base**, keep the base and **subtract** the indices.
- $(a^m)^n = a^{m \times n}$ — When **raising** a **power** to a **power**, keep the base and **multiply** the indices.
- $(ab)^n = a^n b^n$ — The power of a product is the product of the powers.
- $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}, \quad b \neq 0$ — The power of a quotient is the quotient of the powers.

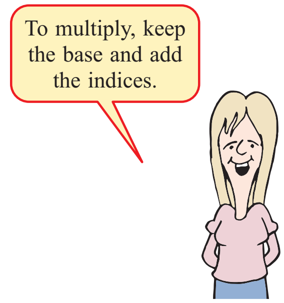
Example 5 — Self Tutor

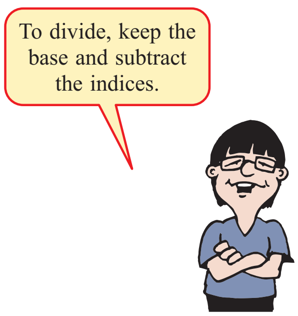
Simplify using the laws of indices:

a $2^3 \times 2^2$ **b** $x^4 \times x^5$

a $2^3 \times 2^2 = 2^{3+2}$
$= 2^5$
$= 32$

b $x^4 \times x^5 = x^{4+5}$
$= x^9$





Example 6 — Self Tutor

Simplify using the index laws: **a** $\dfrac{3^5}{3^3}$ **b** $\dfrac{p^7}{p^3}$

a $\dfrac{3^5}{3^3} = 3^{5-3}$
$= 3^2$
$= 9$

b $\dfrac{p^7}{p^3} = p^{7-3}$
$= p^4$

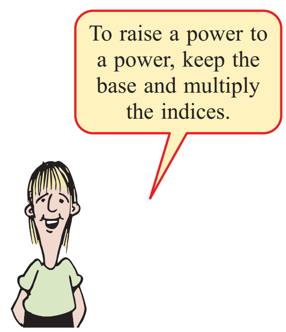
Example 7 — Self Tutor

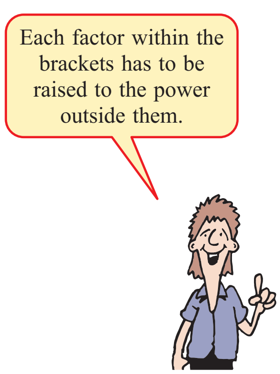
Simplify using the index laws:

a $(2^3)^2$ **b** $(x^4)^5$

a $(2^3)^2$
$= 2^{3\times 2}$
$= 2^6$
$= 64$

b $(x^4)^5$
$= x^{4\times 5}$
$= x^{20}$





Example 8 — Self Tutor

Remove the brackets of:

a $(3a)^2$ **b** $\left(\dfrac{2x}{y}\right)^3$

a $(3a)^2$
$= 3^2 \times a^2$
$= 9a^2$

b $\left(\dfrac{2x}{y}\right)^3$
$= \dfrac{2^3 \times x^3}{y^3}$
$= \dfrac{8x^3}{y^3}$

Example 9 Self Tutor

Express the following in simplest form, without brackets:

a $(3a^3b)^4$ **b** $\left(\frac{x^2}{2y}\right)^3$

a
$$\begin{aligned}&(3a^3b)^4\\&= 3^4 \times (a^3)^4 \times b^4\\&= 81 \times a^{3\times4} \times b^4\\&= 81a^{12}b^4\end{aligned}$$

b
$$\begin{aligned}&\left(\frac{x^2}{2y}\right)^3\\&= \frac{(x^2)^3}{2^3 \times y^3}\\&= \frac{x^{2\times3}}{8 \times y^3}\\&= \frac{x^6}{8y^3}\end{aligned}$$

EXERCISE 6B

1 Simplify using the index laws:

a $2^3 \times 2^1$ **b** $2^2 \times 2^2$ **c** $3^5 \times 3^4$ **d** $5^2 \times 5^3$
e $x^2 \times x^4$ **f** $a^3 \times a$ **g** $n^4 \times n^6$ **h** $b^3 \times b^5$

2 Simplify using the index laws:

a $\frac{2^4}{2^3}$ **b** $\frac{3^5}{3^2}$ **c** $\frac{5^7}{5^3}$ **d** $\frac{4^9}{4^5}$
e $\frac{x^6}{x^3}$ **f** $\frac{y^7}{y^4}$ **g** $a^8 \div a^7$ **h** $b^9 \div b^5$

3 Simplify using the index laws:

a $(2^2)^3$ **b** $(3^4)^3$ **c** $(2^3)^6$ **d** $(10^2)^5$
e $(x^3)^2$ **f** $(x^5)^3$ **g** $(a^5)^4$ **h** $(b^6)^4$

4 Simplify using the index laws:

a $a^5 \times a^2$ **b** $n^3 \times n^5$ **c** $a^7 \div a^3$ **d** $a^5 \times a$
e $b^9 \div b^4$ **f** $(a^3)^6$ **g** $a^n \times a^5$ **h** $(b^2)^4$
i $b^6 \div b^3$ **j** $m^4 \times m^3 \times m^7$ **k** $(a^3)^3 \times a$ **l** $(g^2)^4 \times g^3$

5 Remove the brackets of:

a $(ab)^3$ **b** $(ac)^4$ **c** $(bc)^5$ **d** $(abc)^3$
e $(2a)^4$ **f** $(5b)^2$ **g** $(3n)^4$ **h** $(2bc)^3$
i $\left(\frac{2}{p}\right)^3$ **j** $\left(\frac{a}{b}\right)^3$ **k** $\left(\frac{m}{n}\right)^4$ **l** $\left(\frac{2c}{d}\right)^5$

6 Express the following in simplest form, without brackets:

a $(2b^4)^3$ **b** $\left(\frac{3}{x^2y}\right)^2$ **c** $(5a^4b)^2$ **d** $\left(\frac{m^3}{2n^2}\right)^4$

e $\left(\frac{3a^3}{b^5}\right)^3$ **f** $(2m^3n^2)^5$ **g** $\left(\frac{4a^4}{b^2}\right)^2$ **h** $(5x^2y^3)^3$

C ZERO AND NEGATIVE INDICES [1.9, 2.4]

Consider $\frac{2^3}{2^3}$ which is obviously 1.

Using the exponent law for division, $\frac{2^3}{2^3} = 2^{3-3} = 2^0$

We therefore conclude that $2^0 = 1$.

In general, we can state the **zero index law**: $\boldsymbol{a^0 = 1}$ for all $\boldsymbol{a \neq 0}$.

Now consider $\frac{2^4}{2^7}$ which is $\frac{\cancel{2 \times 2 \times 2 \times 2}^{\,1}}{2 \times 2 \times 2 \times \cancel{2 \times 2 \times 2 \times 2}_{\,1}} = \frac{1}{2^3}$

Using the exponent law of division, $\frac{2^4}{2^7} = 2^{4-7} = 2^{-3}$

Consequently, $2^{-3} = \frac{1}{2^3}$, which means that 2^{-3} and 2^3 are **reciprocals** of each other.

In general, we can state the **negative index law**:

> If a is any non-zero number and n is an integer, then $\boldsymbol{a^{-n} = \frac{1}{a^n}}$.
>
> This means that a^n and a^{-n} are **reciprocals** of one another.
>
> In particular notice that $\boldsymbol{a^{-1} = \frac{1}{a}}$.

Using the negative index law,

$$\left(\tfrac{2}{3}\right)^{-4} = \frac{1}{\left(\frac{2}{3}\right)^4} = 1 \div \frac{2^4}{3^4} = 1 \times \frac{3^4}{2^4} = \left(\tfrac{3}{2}\right)^4$$

So, in general we can see that: $\boldsymbol{\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n}$ provided $a \neq 0$, $b \neq 0$.