1 Indices

1.3 Index Notation

1. Write in a form using indices:

(a) $2 \times 2 \times 2 \times 2$ (b) $3 \times 3 \times 3$

(c) $6 \times 6 \times 6 \times 6 \times 6$ (d) $7 \times 7 \times 7 \times 7 \times 7 \times 7$

(e) $1 \times 1 \times 1 \times 1$ (f) $2 \times 2 \times 2 \times 5 \times 5$

(g) $17 \times 17 \times 17 \times 17$ (h) $5 \times 5 \times 5 \times 6 \times 6 \times 6$

(i) $3 \times 3 \times 7 \times 7 \times 7 \times 7 \times 7$ (j) $2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5$

(k) $5 \times 3 \times 3 \times 3 \times 5$ (l) $11 \times 11 \times 11 \times 11 \times 13 \times 13$

2. Find the value of the following:

(a) 7^2 (b) 3^3 (c) 3^5

(d) 2^8 (e) 7^0 (f) 5^3

(g) 4^3 (h) 6^3 (i) 1^7

(j) 10^6 (k) 2^{10} (l) 3^6

3. Simplify each of the following, leaving your answer in index notation.

(a) $2^7 \times 2^5$ (b) $3^2 \times 3^4 \times 3^6$ (c) $5^2 \times 5^2 \times 5^2$

(d) $4^3 \times 4^7$ (e) $7^2 \times 7^4 \times 7^3$ (f) $2^1 \times 2^5 \times 2^2 \times 2^1$

(g) $2^3 \times 2^3 \times 5^1 \times 5^3$ (h) $3^2 \times 3^3 \times 4^5 \times 4^2 \times 3^1$ (i) $5^2 \times 5^4 \times 5^7$

4. Simplify each of the following, leaving your answer in index notation.

(a) $3^4 \div 3^3$ (b) $7^5 \div 7^2$ (c) $\left(2^3 \times 2^5\right) \div 2^8$

(d) $8^{12} \div 8^7$ (e) $\left(4^3 \times 4^3\right) \div 4^3$ (f) $2^6 \div \left(2^3 \times 2^2\right)$

(g) $\left(9^{10} \times 9^4\right) \div 9^6$ (h) $\left(6^{11} \div 6^{10}\right) \times 6^3$ (i) $\left(10^9 \times 10^9\right) \div 10^{16}$

(j) $\left(4^7 \div 4^3\right) \times 4^2$ (k) $\left(2^1 \times 2^2 \times 2^3\right) \div 2^4$ (l) $6^8 \div \left(6^1 \times 6^2 \times 6^3\right)$

5. Express each of the following numbers as a number to a power, e.g. $256 = 2^8$.

(a) 1024 (b) 243 (c) 125 (d) 216

(e) 512 (f) 169 (g) 343 (h) 1000

(i) 625 (j) 2048 (k) 289 (l) 1331

6. Fill in the missing numbers.

(a) $\left(2^3\right)^2 = 2^{?}$ (b) $\left(3^3\right)^3 = 3^{?}$ (c) $\left(5^{?}\right)^3 = 5^{12}$

(d) $\left(2^4\right)^{?} = 2^8$ (e) $\left(5^3\right)^2 = 5^{?}$ (f) $\left(4^3\right)^{?} = 4^{15}$

(g) $\left(10^3\right)^{?} = 10^9$ (h) $\left(7^4\right)^2 = 7^{?}$ (i) $\left(2^{?}\right)^6 = 2^{12}$

(j) $\left(3^{?}\right)^7 = 3^{21}$ (k) $\left(2^4\right)^{?} = 2^{16}$ (l) $\left(6^{?}\right)^4 = 6^{20}$

7. Simplify the following expressions, leaving your answers in index notation.

(a) $a^4 \times a^3$ (b) $x^5 \div x^2$ (c) $\left(b^4 \times b^3\right) \div b^5$

(d) $a^4 \div a^3$ (e) $x^4 \times x^5$ (f) $\left(x^4 \times x^5\right)^2$

(g) $\left(a^5 \div a^2\right) \times a$ (h) $\left(a^3\right)^2 \times \left(a^2\right)^3$ (i) $\left(x^2 \times x^3\right)^2 \div x^4$

(j) $\left(b^4 \div b^2\right)^3$ (k) $\left(b^4\right)^3 \div \left(b^2\right)^3$ (l) $\left[a^4 \times \left(a^2\right)^3\right] \div a^8$

(m) $\dfrac{x^7 \times x^2}{x^4}$ (n) $\dfrac{a^4 \times \left(a^2\right)^2}{a^8}$ (o) $\dfrac{x^5}{x^2 \times x^2}$

8. (a) Fill in the missing number.

$$2^4 \times 2^{?} = 2^{10}$$

(b) 2^{10} is approximately equal to 1000.
1 000 000 is approximately equal to $2^{?}$

(SEG)

9. Write as a power of 7: (i) $7^3 \times 7^4$ (ii) $7^{11} \div 7^5$ *(Edexcel)*

1.5 Prime Factors

1. Which of the numbers 2, 3, 5, 7, 11, 13 are prime factors of the following numbers?

(a)	189	(b)	264	(c)	490	(d)	770
(e)	165	(f)	288	(g)	2873	(h)	2310
(i)	3640	(j)	6006	(k)	1925	(l)	1815

2. Use the S*ieve of Eratosthenes* to find all prime numbers between 100 and 200.

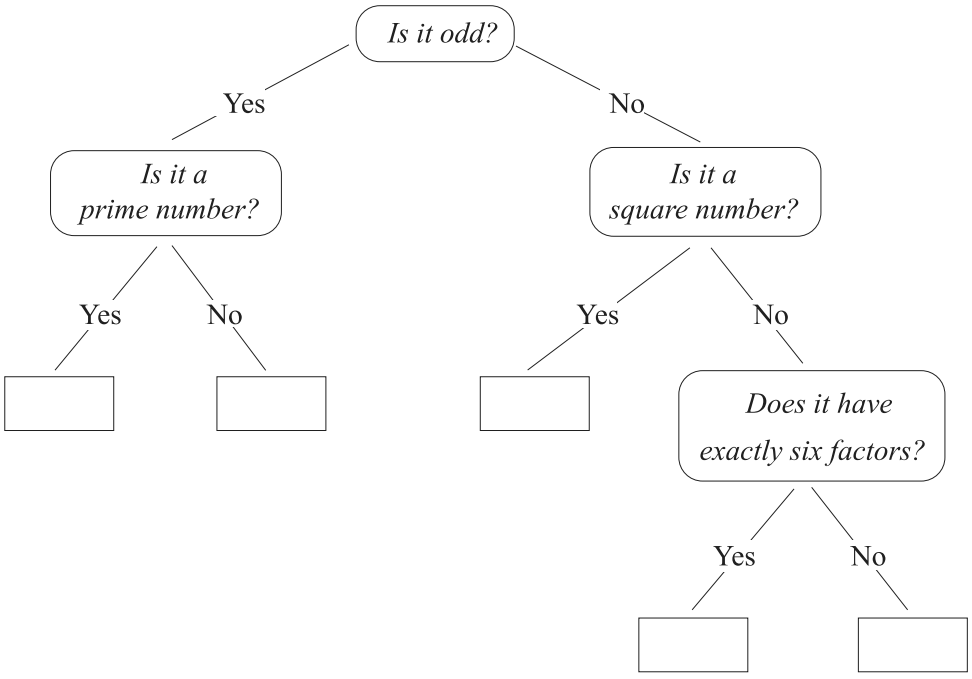
3. Express the following numbers as products of prime numbers.

(a)	150	(b)	60	(c)	72	(d)	144
(e)	315	(f)	210	(g)	284	(h)	180
(i)	270	(j)	231	(k)	306	(l)	500
(m)	702	(n)	3234	(o)	8008	(p)	8190

4. Find the highest common factor of the following.

(a)	16 and 24	(b)	45 and 63	(c)	56 and 70
(d)	90 and 126	(e)	42, 66 and 78	(f)	84, 98 and 154
(g)	189 and 84	(h)	315 and 720	(i)	616 and 392
(j)	560, 140 and 224	(k)	132, 156 and 180	(l)	525, 1400 and 315

5. (a) Copy and put {9, 17, 28, 30} into the correct boxes.



(b) Write down a number that could go into the empty box.

(SEG)

6. (a) Express the following numbers as products of their prime factors.

(i) 72 (ii) 80

(b) Two cars go round a race track. The first car takes 1 minute 12 seconds to complete a circuit and the second car takes 1 minute 20 seconds.

They start level.

Find the length of time before they are next level with one another.

(SEG)

7. (a) Write 18 as the product of its prime factors.

(b) What is the least common multiple (LCM) of 12 and 18?

(AQA)

1.6 Further Index Notation

1. *Without* using a calculator, find the value of each of the following.

(a) $49^{\frac{1}{2}}$ (b) $27^{\frac{1}{3}}$ (c) $16^{\frac{1}{4}}$ (d) $8^{\frac{1}{3}}$

(e) $100^{\frac{1}{2}}$ (f) $9^{1.5}$ (g) $16^{\frac{3}{4}}$ (h) $125^{\frac{2}{3}}$

(i) $32^{\frac{4}{5}}$ (j) $36^{\frac{1}{2}}$ (k) $64^{\frac{1}{2}}$ (l) $121^{\frac{1}{2}}$

(m) $27^{\frac{2}{3}}$ (n) $81^{\frac{3}{4}}$ (o) $1000^{\frac{2}{3}}$ (p) $32^{0.6}$

(q) $4^{1.5}$ (r) $4^{2.5}$ (s) $81^{0.25}$ (t) 5^{-1}

(u) $\left(\frac{1}{4}\right)^{-2}$ (v) $8^{-\frac{1}{3}}$ (w) $9^{-0.5}$ (x) $16^{-0.25}$

(y) $32^{-0.8}$ (z) $16^{-1.5}$

2. What is the value of each of the following expressions? *Do not use a calculator.*

(a) $\left(\frac{1}{4}\right)^{\frac{1}{2}}$ (b) $\left(\frac{1}{8}\right)^{-\frac{2}{3}}$ (c) $(25)^{-2.5}$

(d) $\left(\frac{1}{16}\right)^{-\frac{1}{4}}$ (e) 7^0 (f) $\left(\frac{4}{5}\right)^{-2}$

(g) $\left(\frac{4}{7}\right)^{-2}$ (h) $5^{-2} \times 4^3$ (i) $2^{-3} \div 3^{-3}$

(j) $3^4 \div \left(\frac{1}{3}\right)^{-4}$ (k) $\left(\frac{4}{9}\right)^{-2} \times \left(\frac{27}{8}\right)^{-3}$ (l) $78^{-1} \times 13^3$

3. Express each of the following in index form.

(a) $\left(a^{16}\right)^{\frac{1}{2}}$ (b) $\left(a^{15}\right)^{\frac{1}{3}}$ (c) $\left(27a^9\right)^{\frac{1}{3}}$ (d) $\left(x^{32}\right)^{\frac{1}{4}}$

(e) $\left(x^{20}\right)^{\frac{1}{4}}$ (f) $a^{-2} \div a^{-4}$ (g) $a^4 \times a^{-5}$ (h) $\left(a^2\right)^{-\frac{1}{2}}$

(i) $\left(a^{-4}\right)^2$ (j) $a^{-4} \times a^4$ (k) $\left(a^{\frac{1}{2}}\right)^4$ (l) $\left(a^{16}\right)^{-\frac{1}{4}}$

4. Solve the following equations for x.

(a) $3^x = 81$ (b) $4^x = 64$ (c) $5^x = 125$

(d) $7^x = 49$ (e) $2^x = 32$ (f) $2^x = 64$

(g) $x^3 = 27$ (h) $x^3 = 64$ (i) $x^5 = 32$

(j) $5x^2 = 45$ (k) $3x^3 = 24$ (l) $4x^4 = 324$

(m) $5x^3 = 320$ (n) $x^{\frac{1}{2}} = 3$ (o) $x^{-4} = 256$

(p) $x^{-\frac{1}{2}} = 5$ (q) $2^x = 1$ (r) $3^{-x} = 1$