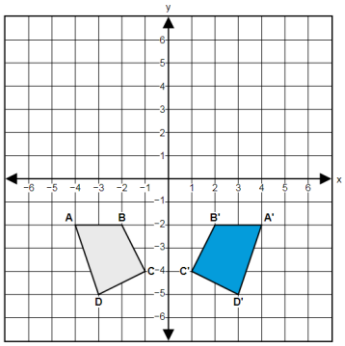
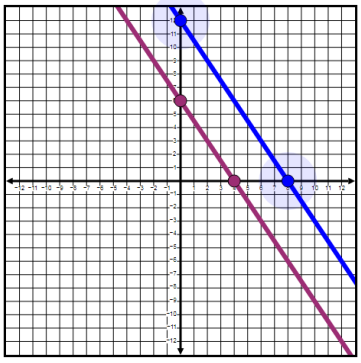
Released Test Answer and Alignment Document

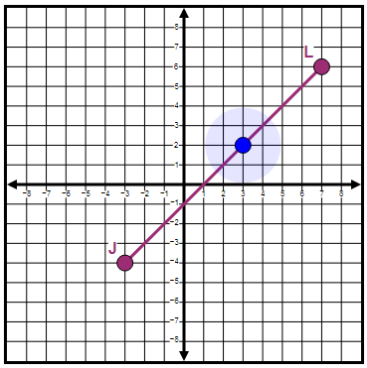
Mathematics – Geometry

Spring 2018

Item Number	Answer Key	Evidence Statement Key
1.	B	G-CO.3
2.	Part A:  Part B: C	G-CO.5
3.	A, C, D, G	G-SRT.2
4.	B, D, E, F	G-CO.1
5.	D, E	G-GMD.4
6.	D	G-GPE.6
7.	Statement / True / False $\triangle ABC \cong \triangle XYZ$ — True $\triangle DEF \cong \triangle XYZ$ — True $\triangle GHJ \cong \triangle XYZ$ — False	G-CO.6
8.	B	G-CO.5
9.	B, E	G-SRT.1b
10.	Part A:	G-GPE.1-2

<table>
<tr><td></td><td>$(x-\boxed{2})^2+(y-\boxed{3})^2=\boxed{13}$

Note:
• The point is earned for any equivalent expression.
Part B:
The center for circle Q is [below] and to the [left] of the center of circle P. The radius of circle Q is [greater than] the radius of circle P.</td><td></td></tr>
<tr><td>11.</td><td>Part A: B, C
Part B:<table><tr><th>Triangle</th><th>Must Be Isosceles</th><th>Must Be Right</th></tr><tr><td>$\triangle ABD$</td><td>☐</td><td>☑</td></tr><tr><td>$\triangle AOD$</td><td>☑</td><td>☐</td></tr><tr><td>$\triangle ABO$</td><td>☑</td><td>☐</td></tr></table></td><td>G-C.2</td></tr>
<tr><td>12.</td><td>Part A:
29
Part B:
76
Part C: A
Part D: C</td><td>G-SRT.8</td></tr>
<tr><td>13.</td><td>See Rubric</td><td>HS-D.3-4a</td></tr>
<tr><td>14.</td><td>Part A: B
Part B: C</td><td>G-SRT.8</td></tr>
<tr><td>15.</td><td>A, C</td><td>G-SRT.1a</td></tr>
<tr><td>16.</td><td>See Rubric</td><td>HS-D.2-2</td></tr>
<tr><td>17.</td><td>  </td><td>G-SRT.1a</td></tr>
</table>

18.		G-GPE.6
19.	D	G-SRT.8
20.	20	G-SRT.5
21.	A, C, E	G-CO.6
22.	See Rubric	HS-C.18.2
23.	The figures in pair 1 [are not] similar because [Triangle ABD is not the image of Triangle CBD after any sequence of similarity t] . The figures in pair 2 [are not] similar because [Triangle EHI is not the image of Triangle GHF after any sequence of similarity tr] . The figures in pair 3 [are] similar because [JKPO is the image of JLMN after a dilation] .	G-SRT.2
24.	sin $p°$= cos [j] ° cos $k°$= sin [v] °	G-SRT.7-2
25.	Part A: The construction is creating an angle congruent to [∠RJQ] . Part B: C	G-CO.D
26.	See Rubric	HS-C.14.1
27.	Part A:	G-Int.1

	The volume of 3 pennies is [greater] than the volume of 1 presidential dollar by approximately [223.1] cubic millimeters. Part B: The amount of cardboard needed to make the box for Arrangement S is [greater than the] amount of cardboard needed for Arrangement Q. The volume of Box S is [the same as] the volume of Box Q. Part C: A Part D: C	
28.	B	G-C.B
29.	C	G-GPE.1-1
30.	See Rubric	HS-C.13.2
31.	Part A: [70.6] Part B: [68]	G-Int.1

#13 Rubric M46629	
Score	**Description**
3	Student response includes the following 3 elements. • **Modeling component** = 2 points ○ Correct reasonable estimate for the height of the building with explanation ○ Valid work or explanation for the length of the arm of the bucket truck • **Computation component** = 1 point ○ Correct reasonable length of the arm of the bucket truck Sample Student Response: If the door is 7.5 feet and it's about 3 sheets of metal siding tall, then I estimated the height of 3 sheets as follows.

	7.5 feet = 90 inches 90 inches divided by 3 sections is about 30 inches per section. I counted the sections on the building, adding one section for the windows at the top and one more section on top of the windows for a total of about 23 sections. This is 23(30) or about 690 inches or about 57.5 feet. The height of the bucket arm is 6 feet off the ground. That would leave 57.5-6 = 51.5 feet of height the arm would need to reach. A model can be a right triangle formed by the bucket arm, a line parallel to the ground, but up 6 feet, and the vertical height off the truck of the bucket arm. The bucket arm length, a, can be estimated using a sin function. $\sin 70 = \frac{51.5}{a}$ $a \sin 70 = 51.5$ $a = \frac{51.5}{\sin 70}$ $a = 54.8$ The arm would need to be at least 54.8 feet to reach the top of the building. The minimum arm length on the bucket truck the repairman should rent to reach the top of the building would be 55 feet.
2	Student response includes 2 of the 3 elements.
1	Student response includes 1 of the 3 elements.
0	Student response is incorrect or irrelevant.

#16 Rubric M47349	
Score	**Description**
3	Student response includes the following 3 elements. • **Modeling component** = 2 points ○ Correct expression that represents the total area of the logo in terms of the radius (equivalent to $\pi r^2 + 14.5$) ○ Valid work or explanation for the value of the radius • **Computation component** = 1 point ○ Correct approximate radius of the inner circle, 5.98 inches Sample Student Response:

	The area of the logo is the sum of the area of the inner circle and the area of the border. The area of the inner circle is πr^2 and the area of the border is given as 14.5 square inches. Therefore, the expression is $\pi r^2 + 14.5$. If the area of the logo is 127 square inches, the radius of the inner circle is approximately 5.98 inches. $\pi r^2 + 14.5 = 127$ $\pi r^2 = 112.5$ $r^2 = \frac{112.5}{\pi}$ $r \approx 5.98$ Notes: • If the radius of the entire logo is found a maximum score of 1 may be awarded. • Radius answers larger than 12 are unreasonable and do not receive credit.
2	Student response includes 2 of the 3 elements.
1	Student response includes 1 of the 3 elements.
0	Student response is incorrect or irrelevant.

#22 Rubric VH184676	
Score	**Description**
4	Student response includes the following 4 elements. • **Reasoning component** = 4 points ○ Correct calculation of the four midpoints ○ Correctly shows that segments EF and GH are parallel ○ Correctly shows that segments FG and GH are parallel ○ Correctly concludes that EFGH is a parallelogram Sample Student Response: "E" Midpoint of $\overline{AB}$: $\left(\frac{-2+4}{2}, \frac{3+5}{2}\right) = (1,4)$ "F" Midpoint of $\overline{BC}$: $\left(\frac{4+10}{2}, \frac{5-1}{2}\right) = (7,2)$ "G" Midpoint of $\overline{CD}$: $\left(\frac{10+8}{2}, \frac{-1-9}{2}\right) = (9,-5)$ "H" Midpoint of $\overline{DA}$: $\left(\frac{8-2}{2}, \frac{-9+3}{2}\right) = (3,-3)$

Slope of $\overline{EF}$: $\frac{2-4}{7-1} = \frac{-2}{6} = -\frac{1}{3}$

Slope of $\overline{GH}$: $\frac{-3-(-5)}{3-9} = \frac{2}{-6} = -\frac{1}{3}$

Slope of $\overline{FG}$: $\frac{-5-2}{9-7} = \frac{-7}{2} = -\frac{7}{2}$

Slope of $\overline{EH}$: $\frac{-3-4}{3-1} = \frac{-7}{2} = -\frac{7}{2}$

Because both pairs of opposite sides are parallel, quadrilateral EFGH is a parallelogram.

Notes:

- There are other ways to show that quadrilateral EFGH is a parallelogram.
    - Show that segments EF and GH are congruent, and that segments FG and EH are congruent.

$$\overline{EF} = \sqrt{(7-1)^2 + (2-4)^2} = \sqrt{6^2 + (-2)^2} = \sqrt{36+4} = \sqrt{40}$$
$$\overline{GH} = \sqrt{(3-9)^2 + (-3-(-5))^2} = \sqrt{(-6)^2 + 2^2} = \sqrt{36+4} = \sqrt{40}$$
$$\overline{FG} = \sqrt{(9-7)^2 + (-5-2)^2} = \sqrt{2^2 + (-7)^2} = \sqrt{4+49} = \sqrt{53}$$
$$\overline{EH} = \sqrt{(3-1)^2 + (-3-4)^2} = \sqrt{2^2 + (-7)^2} = \sqrt{4+49} = \sqrt{53}$$

    - Show that segments EF and GH are congruent and parallel.
    - Show that segments FG and EH are congruent and parallel. (For either of these, use the appropriate lines from above.)
    - Show that the diagonals bisect each other.

Midpoint of $\overline{EG}$: $\left(\frac{1+9}{2}, \frac{4-5}{2}\right) = \left(5, -\frac{1}{2}\right)$

Midpoint of $\overline{FH}$: $\left(\frac{7+3}{2}, \frac{2-3}{2}\right) = \left(5, -\frac{1}{2}\right)$

	• For any valid pair of facts and associated conclusion, award full credit. ○ Note that it is NOT enough to show that segments EF and GH are congruent, while segments FG and EH are parallel. If a student does this, he/she earns 3 out of 4 points. The same holds true if the student shows that segments EF and GH are parallel, while segments FG and EH are congruent.
3	Student response includes 3 of the 4 elements.
2	Student response includes 2 of the 4 elements.
1	Student response includes 1 of the 4 elements.
0	Student response is incorrect or irrelevant.

#26 Rubric VF671199	
Score	**Description**
3	The response shows complete understanding of congruent triangles and the properties needed to prove that the lines are perpendicular. The proof contains logical steps and justification for each step to prove the lines are perpendicular. Sample Student Response:

	<table><tr><th>Statement</th><th>Reason</th></tr><tr><td>$AB = CB$</td><td>Given</td></tr><tr><td>$\overline{AB} \cong \overline{CB}$</td><td>Definition of congruent segments</td></tr><tr><td>D is the midpoint of $\overline{AC}$</td><td>Given</td></tr><tr><td>$\overline{BD}$ bisects $\overline{AC}$</td><td>Definition of segment bisector</td></tr><tr><td>$\overline{AD} \cong \overline{DC}$</td><td>Definition of midpoint</td></tr><tr><td>$\overline{BD} \cong \overline{BD}$</td><td>Reflexive property of segment congruence</td></tr><tr><td>$\Delta ADB \cong \Delta CDB$</td><td>SSS Congruence Postulate</td></tr><tr><td>$\angle ADB \cong \angle CDB$</td><td>Corresponding parts of congruent triangles are congruent. (CPCTC)</td></tr><tr><td>$m\angle ADB = m\angle CDB$</td><td>Definition of congruent angles</td></tr><tr><td>$m\angle ADB + m\angle CDB = 180°$</td><td>Definition of a straight angle</td></tr><tr><td>$m\angle ADB + m\angle ADB = 180°$</td><td>Substitution</td></tr><tr><td>$m\angle ADB = 90°$</td><td>Division property of equality</td></tr><tr><td>$\angle ADB$ is a right angle</td><td>Definition of a right angle</td></tr><tr><td>$\overline{AC} \perp \overline{BD}$</td><td>Definition of perpendicular lines</td></tr></table> Note: ○ Credit can be earned for providing triangles congruent ($\Delta ADB \cong \Delta CDB$) with valid reasoning. ○ Credit can be earned for proving $\overline{AC} \perp \overline{BD}$. with valid reasoning. ○ The proof does not have to be a tow-column proof, but it needs to have the reasoning behind each step.
2	The response shows some understanding of the logical proof process and attempts to write a logical proof using congruent triangles and right angles but makes mistakes logic and steps in justification.
1	The response shows minimal understanding and attempts to write a proof

	using properties of congruent triangles but does not show much ability to write a logical argument with reasons or justification.
0	The response is incorrect or irrelevant.

#30 Rubric M43437	
Score	**Description**
4	Student response includes the following 4 elements. • **Reasoning component** = 4 points ○ Correct coordinates of point Q ○ Valid reasoning for determining the coordinates of point Q ○ Valid argument to justify the conclusion that the length of the median to the hypotenuse of a right triangle is half the length of the hypotenuse ○ Correct calculations for finding expressions for the lengths of line segments that are relevant to the argument Sample Student Response: Line segment *MQ* is a median of Δ*MPR*. This means that, by the definition of median, point *Q* is the midpoint of line segment *PR*. I used the Midpoint Formula to find the coordinates of point *Q*. *x*-coordinate of point Q = $\frac{2a + 0}{2} = \frac{2a}{2} = a$ *y*-coordinate of point *Q* = $\frac{0 + 2b}{2} = \frac{2b}{2} = b$ The coordinates for point *Q* are (*a, b*). The hypotenuse of Δ*MPR* is line segment *PR*. So first I used the Distance Formula to find the length of segment *PR*. Length of line segment *PR* $= \sqrt{(2a - 0)^2 + (0 - 2b)^2} = \sqrt{4a^2 + 4b^2} = 2\sqrt{a^2 + b^2}$ This means I have to show that the length of line segment *MQ* is half of $2\sqrt{a^2 + b^2}$ which equals $\sqrt{a^2 + b^2}$ Length of line segment *MQ*

	$= \sqrt{(a - 0)^2 + (b - 0)^2} = \sqrt{a^2 + b^2}$ I showed that the length of line segment *MQ* is half the length of line segment *PR*. This proves that the length of the median from the right angle to the hypotenuse of a right triangle is half the length of the hypotenuse.
3	Student response includes 3 of the 4 elements.
2	Student response includes 2 of the 4 elements.
1	Student response includes 1 of the 4 elements.
0	Student response is incorrect or irrelevant.