15 Two bushwalkers set off from base camp at the same time, walking at right angles to one another. One walks at an average speed of 5 km/h, and the other at an average speed of 4 km/h. Find their distance apart after 3 hours.

16 To get to school from her house, Ella walks down Bernard Street, then turns 90° and walks down Thompson Road until she reaches her school gate. She walks twice as far along Bernard Street as she does along Thompson Road. If Ella's house is 2.5 km in a straight line from her school gate, how far does Ella walk along Bernard Street?

17 Boat A is 10 km east of boat B. Boat A travels 6 km north, and boat B travels 2 km west. How far apart are the boats now?

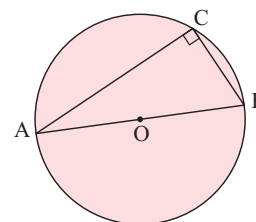## D   CIRCLE PROBLEMS   [4.6, 4.7]

There are certain properties of circles which involve right angles. In these situations we can apply Pythagoras' theorem. The properties will be examined in more detail in **Chapter 27**.

ANGLE IN A SEMI-CIRCLE

> The angle in a semi-circle is a right angle.

No matter where C is placed on the arc AB, $A\widehat{C}B$ is always a right angle.

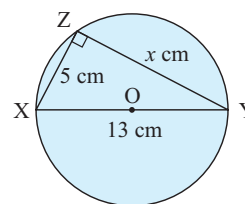### Example 13   ◆) Self Tutor

A circle has diameter XY of length 13 cm. Z is a point on the circle such that XZ is 5 cm. Find the length YZ.

From the angle in a semi-circle theorem, we know $X\widehat{Z}Y$ is a right angle.
Let the length YZ be x cm.

$$\therefore \quad 5^2 + x^2 = 13^2 \qquad \{\text{Pythagoras}\}$$
$$\therefore \quad x^2 = 169 - 25 = 144$$
$$\therefore \quad x = \sqrt{144} \qquad \{\text{as} \quad x > 0\}$$
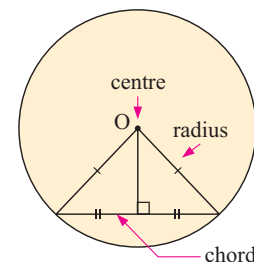$$\therefore \quad x = 12$$

So, YZ has length 12 cm.

A CHORD OF A CIRCLE

> The line drawn from the centre of a circle at right angles to a chord bisects the chord.

This follows from the **isosceles triangle theorem**. The construction of radii from the centre of the circle to the end points of the chord produces two right angled triangles.
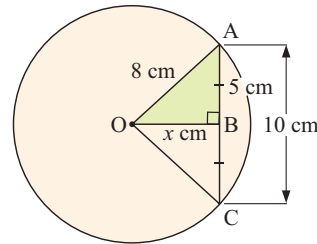
Example 14   ◀》 *Self Tutor*

A circle has a chord of length 10 cm. If the radius of the circle is 8 cm, find the shortest distance from the centre of the circle to the chord.

The shortest distance is the 'perpendicular distance'. The line drawn from the centre of a circle, perpendicular to a chord, bisects the chord, so

$$AB = BC = 5 \text{ cm}.$$

In $\triangle AOB$, $\quad 5^2 + x^2 = 8^2 \qquad$ {Pythagoras}

$$\therefore \quad x^2 = 64 - 25 = 39$$

$$\therefore \quad x = \sqrt{39} \qquad \{\text{as} \quad x > 0\}$$
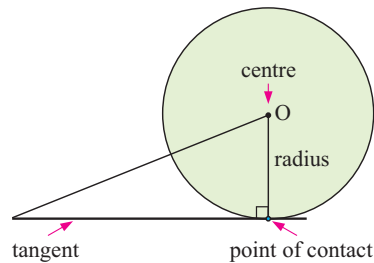
$$\therefore \quad x \approx 6.24$$

So, the shortest distance is about 6.24 cm.

TANGENT-RADIUS PROPERTY

A tangent to a circle and a radius at the point of contact meet at right angles.

Notice that we can now form a right angled triangle.

Example 15   ◀》 *Self Tutor*

A tangent of length 10 cm is drawn to a circle with radius 7 cm. How far is the centre of the circle from the end point of the tangent?
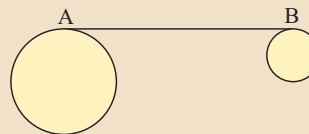
Let the distance be d cm.

$$\therefore \quad d^2 = 7^2 + 10^2 \qquad \{\text{Pythagoras}\}$$

$$\therefore \quad d^2 = 149$$

$$\therefore \quad d = \sqrt{149} \qquad \{\text{as} \quad d > 0\}$$

$$\therefore \quad d \approx 12.2$$
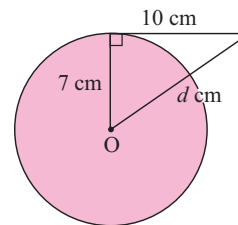
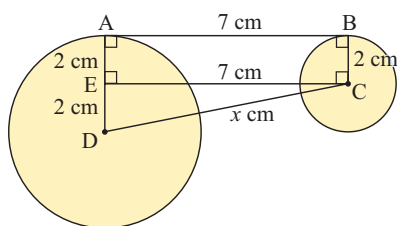So, the centre is 12.2 cm from the end point of the tangent.

Example 16   ◀》 *Self Tutor*

Two circles have a common tangent with points of contact at A and B. The radii are 4 cm and 2 cm respectively. Find the distance between the centres given that AB is 7 cm.

For centres C and D, we draw BC, AD, CD and CE ∥ AB.

∴ ABCE is a rectangle

∴ CE = 7 cm {as CE = AB}

and DE = 4 − 2 = 2 cm

Now $x^2 = 2^2 + 7^2$ {Pythagoras in $\triangle DEC$}

∴ $x^2 = 53$

∴ $x = \sqrt{53}$ {as $x > 0$}
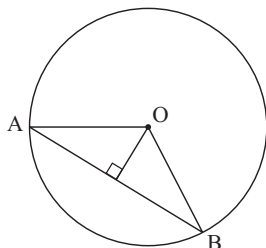
∴ $x \approx 7.28$

∴ the distance between the centres is about 7.28 cm.

EXERCISE 8D

1 AT is a tangent to a circle with centre O. The circle has radius 5 cm and AB = 7 cm. Find the length of the tangent.
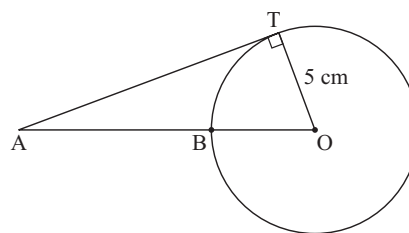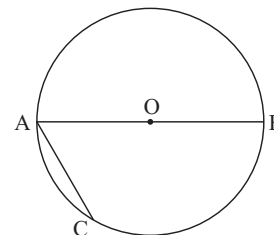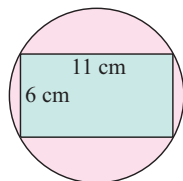
2 A circle has centre O and a radius of 8 cm. Chord AB is 13 cm long. Find the shortest distance from the chord to the centre of the circle.

3 AB is a diameter of a circle and AC is half the length of AB. If BC is 12 cm long, what is the radius of the circle?
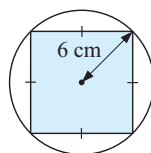
4 A rectangle with side lengths 11 cm and 6 cm is inscribed in a circle. Find the radius of the circle.

5 A circle has diameter AB of length 10 cm. C is a point on the circle such that AC is 8 cm. Find the length BC.

6 A square is inscribed in a circle of radius 6 cm. Find the length of the sides of the square, correct to 3 significant figures.