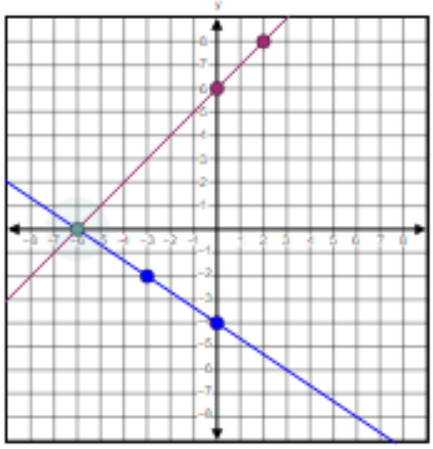
# Released Test Answer and Alignment Document

## Mathematics – Algebra 1

## Performance Based Assessment

The following pages include the answer key for all machine-scored items, followed by the rubrics for the hand-scored items.

- The rubrics show sample student responses. Other valid methods for solving the problem can earn full credit unless a specific method is required by the item.
- In items where the scores are awarded for full and partial credit, the definition of partial credit will be confirmed during range-finding (reviewing sets of real student work).
- If students make a computation error, they can still earn points for reasoning or modeling.

| Item Number | Answer Key | Evidence Statement Key | Integrated Course Alignment |
|---|---|---|---|
| 1. | B, D | F-IF.1 | Math 1 |
| 2. | B | F-IF.6-6a | |
| 3. | $x = -7$<br>$x = 2$ | A-REI.4b-1 | Math 2 |
| 4. |  | A-REI.11-1a | |
| 5. | $a =$ 2<br>$b =$ 2<br>$c =$ -12 | A-APR.1-1 | Math 2 |
| 6. | A | A-CED.4-1 | Math 1 |

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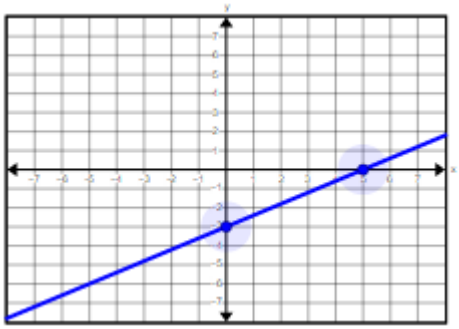
|---|---|---|---|
| 7. | $(x$ - 1 $)(x$ - 3 $)$<br>or<br>$(x$ - 3 $)(x$ - 1 $)$<br>$x =$ 1 ; $x =$ 3 | A-SSE.3a | Math 2 |
| 8. | D | A-REI.10 | |
| 9. | 25.12 square feet per foot | F-IF.6-1a | |
| 10. |  | F-IF.7a-1 | Math 1 |
| 11. | Part A: see rubric<br>Part B: see rubric | HS.D.2-6 | Math 2 |
| 12. | Part A: see rubric<br>Part B: see rubric | HS.D.2-9 | Math 2 |
| 13. | See rubric | HS.C.2.1 | Math 2 |
| 14. | Part A: see rubric<br>Part B: see rubric<br>Part C: see rubric | HS.D.1-1 | Math 1 |
| 15. | Part A: see rubric<br>Part B: see rubric | HS.C.10.1 | Math 1 |
| 16. | Part A: see rubric<br>Part B: see rubric | HS.C.18.1 | Math 1 |
| 17. | Part A: see rubric<br>Part B: see rubric | HS.C.16.2 | Math 2 |
| 18. | See rubric | HS.D.3-1 | Math 1 |

| #11 Part A | |
|---|---|
| **Score** | **Description** |
| **2** | Student response includes the following 2 elements.<br>• **Modeling component** = 2 points<br>o Valid equation for width of the rug<br>o Valid explanation for the equation<br>Sample Student Response:<br>$w(w + 4) = 21$, where w is the width of rug<br>The area of the rug is $A = lw$, which is 21 square feet. The length is $l = w + 4$. Replacing l with $w + 4$ and A with 21 gives the equation $w(w + 4) = 21$. |
| **1** | Student response includes 1 of the 2 elements. |
| **0** | Student response is incorrect or irrelevant. |

| #11 Part B | |
|---|---|
| **Score** | **Description** |
| **4** | Student response includes the following 4 elements.<br>• **Modeling component** = 3 points<br>o Determination of the original length and width of the rug<br>o Implementation of the 1.5 feet border into the calculation<br>o Work to determine the new area<br>• **Computation component** = 1 point<br>o Correct computation based on work<br>Sample Student Response:<br>$w^2 + 4w - 21 = 0$<br>$(w + 7)(w - 3) = 0$<br>$w + 7 = 0$ OR $w - 3 = 0$<br>$w = -7$ OR $w = 3$<br>Since width cannot be negative, the width is 3 feet. So, the length is 7 feet.<br>The 1.5 foot border would add 3 feet to each dimension. |

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| | A = (3 + 3)(7 + 3)<br>A = (6)(10)<br>A = 60<br>60 square feet<br>Note: A student can earn full credit for the modeling component in Part B without explicitly stating the width and length of the rug if the reasoning is mathematically appropriate. |
| **3** | Student response includes 3 of the 4 elements. |
| **2** | Student response includes 2 of the 4 elements. |
| **1** | Student response includes 1 of the 4 elements. |
| **0** | Student response is incorrect or irrelevant. |

| #12 Part A | |
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| **Score** | **Description** |
| **2** | Student response includes the following 2 elements.<br>• **Modeling component** = 2 points<br>o Valid function<br>o Work to support the function<br>Sample Student Response:<br>f(x) = 6x – 450<br>The total cost of the prizes is 349 + 42 + 25 + 18 + 16 = 450.<br>For 75 tickets to make $450, they must each cost 450 ÷ 75 = $6. |
| **1** | Student response includes 1 of the 2 elements. |
| **0** | Student response is incorrect or irrelevant. |

| #12 Part B | |
|---|---|
| **Score** | **Description** |
| **1** | Student response includes the following element.<br>• **Computation component** = 1 points<br>o Machine Scored: 217 tickets |
| **0** | Student response is incorrect or irrelevant. |

| #13 Rubric | |
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| **Score** | **Description** |
| **3** | Student response includes the following 3 elements.<br>• **Reasoning component** = 3 points<br>o Correct identification of *a* as rational and *b* as irrational<br>o Correct identification that the product is irrational<br>o Correct reasoning used to determine rational and irrational numbers<br><br>Sample Student Response:<br>A rational number can be written as a ratio. In other words, a number that can be written as a simple fraction. $a = 0.444444444444...$ can be written as $\frac{4}{9}$. Thus, *a* is a rational number. All numbers that are not rational are considered irrational. An irrational number can be written as a decimal, but not as a fraction. $b = 0.354355435554...$ cannot be written as a fraction, so it is irrational. The product of an irrational number and a nonzero rational number is always irrational, so the product of *a* and *b* is irrational. You can also see it is irrational with my calculations:<br>$\frac{4}{9}(.354355435554...) = .15749...$<br>$.15749...$ is irrational. |
| **2** | Student response includes 2 of the 3 elements. |
| **1** | Student response includes 1 of the 3 elements. |
| **0** | Student response is incorrect or irrelevant. |

| #14 Part A | |
|---|---|
| **Score** | **Description** |
| **2** | Student response includes the following 2 elements.<br>• **Modeling component** = 2 points<br>o Correct equation for *Office Essentials*<br>o Correct equation for *Everything Office*<br><br>Sample Student Response:<br>$M = 2,500 + 125n$ represents the monthly earnings for the *Office Essentials* job.<br>$M = 2,000 + 150n$ represents the monthly earnings for the *Everything Office* job. |
| **1** | Student response includes 1 of the 2 elements. |
| **0** | Student response is incorrect or irrelevant. |

| #14 Part B | |
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| **Score** | **Description** |
| **2** | Student response includes the following 2 elements.<br>• **Modeling component** = 2 points<br>o For *Office Essentials*, the conclusion that Marcella must sell at least 12 machines per month in order to meet her goal, with supporting work<br>o For *Everything Office*, the conclusion that Marcella must sell at least 14 machines per month in order to meet her goal, with supporting work<br>Sample Student Response:<br>For each company, write and solve an inequality. |

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| | *Office Essentials*<br>$2{,}500 + 125n \geq M$<br>$2{,}500 + 125n \geq 4{,}000$<br>$125n \geq 1{,}500$<br>$n \geq 12$<br><br>*Everything Office*<br>$2{,}000 + 150n \geq M$<br>$2{,}000 + 150n \geq 4{,}000$<br>$150n \geq 2{,}000$<br>$n \geq 13\frac{1}{3}$<br><br>For *Office Essentials*, the result $n \geq 12$ means that Marcella must sell at least 12 office machines per month in order to earn a total of at least \$4,000.<br><br>At *Everything Office*, the result $n \geq 13\frac{1}{3}$ must be interpreted within the context of the problem as $n \geq 14$, since it is impossible to sell one-third of an office machine. So Marcella must sell at least 14 office machines per month in order to earn a total of at least \$4,000. |
| **1** | Student response includes 1 of the 2 elements. |
| **0** | Student response is incorrect or irrelevant. |

| #14 Rubric Part C | |
|---|---|
| **Score** | **Description** |
| **2** | Student response includes the following 2 elements.<br>• **Modeling component** = 2 points<br>o Calculations or reasoning to show that the total earnings at the two companies are equal when 20 machines are sold<br>o The conclusion that the total earnings for the *Everything Office* job are greater than the total earnings for the *Office Essentials* job when the representative sells any number of office machines greater than 20, with supporting work.<br><br>Sample Student Response:<br><br>Use the equations from Part A to form a system of equations.<br><br>$\begin{cases} M = 2{,}500 + 125n \\ M = 2{,}000 + 150n \end{cases}$<br><br>Solve the system by substituting $2{,}000 + 150n$ for $M$ in the first equation and then solving the resulting equation for $n$. |

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| | $$2{,}000 + 150n = 2{,}500 + 125n$$<br>$$2{,}000 + 25n = 2{,}500$$<br>$$25n = 500$$<br>$$n = 20$$<br><br>In both cases, the value of $M$ is 5,000 when 20 is substituted for $n$. So the total earnings at *Everything Office* equal the total earnings at *Office Essentials* when 20 office machines are sold.<br><br>To find the interval of machines sold for which the total earnings at *Everything Office* are greater than the total earnings at *Office Essentials*, I can set up the inequality:<br><br>$$2{,}000 + 150n > 2{,}500 + 125n$$<br>$$25n > 500$$<br>$$n > 20$$<br><br>When Marcella sells more than 20 machines her earnings at *Everything Office* are greater than at *Office Essentials.* |
| **1** | Student response includes 1 of the 2 elements. |
| **0** | Student response is incorrect or irrelevant. |

<table>
<tr><th colspan="2">#15 Part A</th></tr>
<tr><th>Score</th><th>Description</th></tr>
<tr><td>1</td><td>Student response includes the following element.<ul><li>Computation component = 1 point<ul><li>
<table>
<tr><th></th><th>Web Site A</th><th>Web Site B</th></tr>
<tr><td>Week 1:</td><td>25</td><td>25</td></tr>
<tr><td>Week 2:</td><td>50</td><td>175</td></tr>
<tr><td>Week 3:</td><td>100</td><td>325</td></tr>
<tr><td>Week 4:</td><td>200</td><td>475</td></tr>
<tr><td>Week 5:</td><td>400</td><td>625</td></tr>
<tr><td>Week 6:</td><td>800</td><td>775</td></tr>
<tr><td>Week 7:</td><td>1,600</td><td>925</td></tr>
<tr><td>Week 8:</td><td>3,200</td><td>1,075</td></tr>
</table>
</li></ul></li></ul>Note: All values must be correct to earn the point.</td></tr>
<tr><td>0</td><td>Student response is incorrect or irrelevant.</td></tr>
<tr><th colspan="2">#15 Part B</th></tr>
<tr><th>Score</th><th>Description</th></tr>
<tr><td>2</td><td>Student response includes the following 2 elements.<ul><li>Reasoning component = 2 points<ul><li>Valid reasoning about the nature of the data for Web Site A</li><li>Valid reasoning about the nature of the data for Web Site B</li></ul></li></ul>Sample Student Response:<br>Jose’s claim is not correct about Web Site A. A linear model assumes that the rate of change is constant for each one-week increment. Looking at the data in the table, the change from Week 1 to Week 2 is 25, the change from Week 2 to Week 3 is 50, and the change from Week 3 to Week 4 is 100. The change in visitors from one week to the next is not constant; the change in visitors from one</td></tr>
</table>

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| | week to the next is two times the change in the previous week.<br>Such data are better suited for an exponential model.<br>Jose's claim is correct about Web Site B. The rate of change in the number of visitors from one week to the next is constant. The change from one week to the next is 150 visitors each week. |
| **1** | Student response includes 1 of the 2 elements. |
| **0** | Student response is incorrect or irrelevant. |

| #16 Part A | |
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| **Score** | **Description** |
| **2** | Student response includes the following 2 elements.<br>• **Reasoning component** = 1 point<br>o correct explanation that the slope is equal between any two points on a given line and that each slope must equal 2, since that is the slope of the given line<br>• **Computation component** = 1 point<br>o correct calculation of $m = 2$ for the slopes of $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$<br><br>Sample Student Response:<br>Calculate each slope.<br>$\overline{AB}$ :<br>$\frac{(2b-3)-(2a-3)}{b-a} = \frac{2b-3-2a-(-3)}{b-a} = \frac{2b-2a+(-3)-(-3)}{b-a} = \frac{2b-2a+(-3)+3}{b-a} = \frac{2b-2a}{b-a} = \frac{2(b-a)}{b-a} = 2$<br>$\overline{BC}$ : |

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| | $\frac{(2c-3)-(2b-3)}{c-b} = \frac{2c-3-2b-(-3)}{c-b}$<br>$= \frac{2c-2b+(-3)-(-3)}{c-b}$<br>$= \frac{2c-2b+(-3)+3}{c-b}$<br>$= \frac{2c-2b}{c-b}$<br>$= \frac{2(c-b)}{c-b} = 2$<br><br>$\overline{AC}$ :<br><br>$\frac{(2c-3)-(2a-3)}{c-a} = \frac{2c-3-2a-(-3)}{c-a}$<br>$= \frac{2c-2a+(-3)-(-3)}{c-a}$<br>$= \frac{2c-2a+(-3)+3}{c-a}$<br>$= \frac{2c-2a}{c-a}$<br>$= \frac{2(c-a)}{c-a} = 2$<br><br>Each ratio equals 2. Amy is correct. All the slopes are equal.<br><br>Note: A student can also receive a score of 2 by demonstrating a complete understanding, using appropriate mathematical reasoning/work. |
| **1** | Student response includes 1 of the 2 elements.<br><br>Note: A student can also receive a score of 1 by demonstrating a partial understanding, using appropriate mathematical reasoning/work. |
| **0** | Student response is incorrect or irrelevant. |
| #16 Part B | |
| **Score** | **Description** |
| **2** | Student response includes the following 2 elements. |

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| | • **Reasoning component** = 2 points<br>o correct explanation that $(-1, 1)$ is not on the graph of $y = 2x - 3$<br>o correct explanation that $(1, -1)$ is on the graph of $y = 2x - 3$<br><br>Sample Student Response:<br><br>To determine whether a point lies on the graph of an equation, substitute the coordinates of the point into the equation. If the coordinates of the point make the equation true, then the point is on the graph of the equation.<br>$(-1, 1)$<br>$1 = 2(-1) - 3$<br>$1 = -2 - 3$<br>$1 \neq -5$<br><br>Therefore, $(-1, 1)$ is not on the graph.<br><br>$(1, -1)$<br>$-1 = 2(1) - 3$<br>$-1 = 2 - 3$<br>$-1 = -1$<br><br>Therefore, $(1, -1)$ is on the graph. |
| **1** | Student response includes 1 of the 2 elements. |
| **0** | Student response is incorrect or irrelevant. |

| #17 Part A | |
|---|---|
| **Score** | **Description** |
| **2** | Student response includes the following 2 elements.<br>• **Reasoning component** = 1 point<br>o Algebraic or written explanation for solving the equation<br>• **Computation component** = 1 point<br>o Solution of $x = 2$ or -14<br><br>Sample Student Response:<br>$x^2 + 12x - 28 = 0$<br>$x^2 + 12x = 28$<br>$x^2 + 12x + 36 = 28 + 36$<br>$(x + 6)^2 = 64$<br>$x + 6 = \pm 8$<br>$x = 8 - 6 = 2$ or $x = -8 - 6 = -14$<br>Solution 1: 2<br>Solution 2: –14 |
| **1** | Student response includes 1 of the 2 elements. |
| **0** | Student response is incorrect or irrelevant. |

| #17 Part B | |
|---|---|
| **Score** | **Description** |
| **2** | Student response includes the following 2 elements.<br>• **Reasoning component** = 1 point |

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| | o Valid explanation<br><br>• **Computation component** = 1 point<br>o Solution of *c* = 36<br><br>Sample Student Response:<br>There would be only one solution if the factors of the polynomial are the same. If the factors are the same, then the identity $(x + a)^2 = x^2 + 2ax + a^2$ can be used. The middle term is 12, so *c* would have to be the square of half of that number. Therefore c = 36 |
| **1** | Student response includes 1 of the 2 elements. |
| **0** | Student response is incorrect or irrelevant. |

| #18 Rubric | |
|---|---|
| **Score** | **Description** |
| **3** | Student response includes the following 3 elements.<br>• **Modeling component** = 2 points<br>o correct equation, $w \approx 16 - 0.19h$<br>o accurate use of notation and vocabulary to support correct calculations and mathematical reasoning, identifying variables as needed<br>• **Computation component** = 1 point<br>o correct application of the model to make an accurate prediction<br>Sample Student Response:<br>If the burn rate is believed to be constant, determine the average burn rate for the eight candles as the ratio of weight loss per hour.<br>ounces lost over three-hours<br>$\frac{0.5+0.6+0.5+0.7+0.7+0.5+0.5+0.6}{8} \approx 0.575$<br>ounces lost per hour on average<br>$\frac{0.575}{3} \approx 0.19$<br>For 0 hours, the weight of each candle is 16 ounces. Therefore,<br>$w \approx 16 - 0.19h$.<br>This model can be used to predict the weight of the candle when $h$, the number of hours of burning, is 5. |

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| | $w \approx 16 - 0.19(5)$<br>$\approx 16 - 0.95$<br>$\approx 15.05$<br><br>According to the model, the weight of the candle after 5 hours of burning would be about 15.05 ounces. |
| **2** | Student response includes 2 of the 3 elements. |
| **1** | Student response includes 1 of the 3 elements. |
| **0** | Student response is incorrect or irrelevant. |