PARCC

Partnership for Assessment of
Readiness for College and Careers

Math
Spring Operational 2015

Geometry
End of Year Released Items

1. The circle has a radius of 12 units. Shade an area of 24π square units.

Divide the circle into the correct number of sections by selecting the "More" button. If you divide the circle into too many sections, use the "Fewer" button. Then, select the number of sections to represent the answer.

| Fewer | More | Reset |
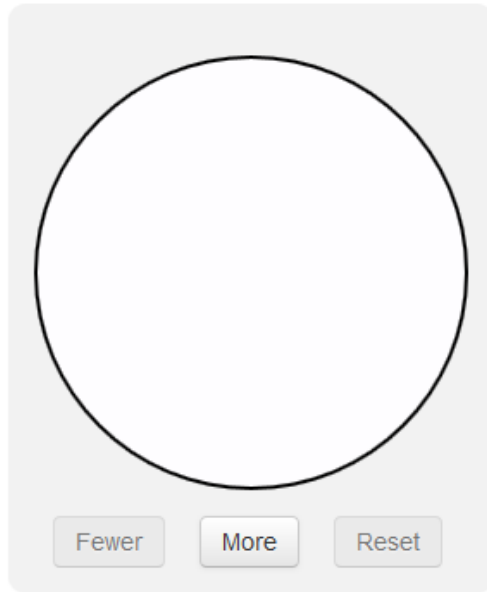
2. Octagon $PQRSTVWZ$ is a regular octagon with its center at point C.
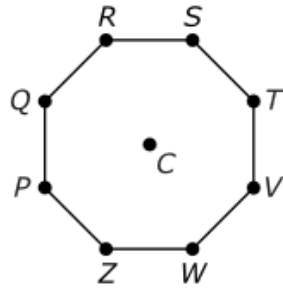
Which transformations will map octagon $PQRSTVWZ$ onto itself?

Select **each** correct transformation.

☐ A. reflecting over $\overline{QV}$

☐ B. reflecting over $\overline{RW}$

☐ C. reflecting over $\overline{TZ}$

☐ D. rotating $45°$ clockwise around point Z

☐ E. rotating $135°$ clockwise around point C

☐ F. rotating $90°$ counterclockwise around point C

3. A dilation with a center at $P(0, 0)$ and a scale factor k is applied to $\overline{MN}$. Let $\overline{M'N'}$ represent the image of $\overline{MN}$ after the dilation.

Select **each** correct statement.

☐   A. If $k > 0$, then $M'N' > MN$.

☐   B. If $k > 1$, then $M'N' > MN$.

☐   C. If $0 < k < 1$, then $M'N' < MN$.

☐   D. If $0.5 < k < 1.5$, then $M'N' < MN$.

☐   E. If $k = 1$, then $M'N' = MN$.

☐   F. If $k = 0.5$, then $M'N' = 0.5(MN)$.

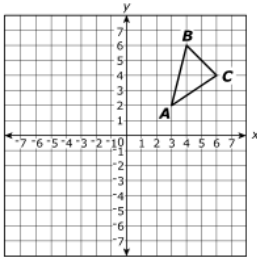4. The equation $x^2 - 10x + 17 = -y^2 - 2y$ describes a circle in the coordinate plane.

Find the radius of the circle and the coordinates of its center.

Enter your answers in the spaces provided. Enter **only** your answers.

radius $=\square$ units   center : $(\square, \square)$

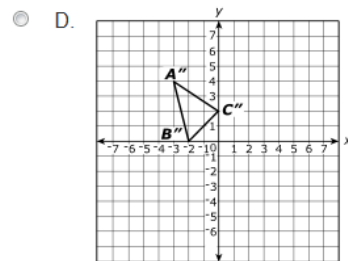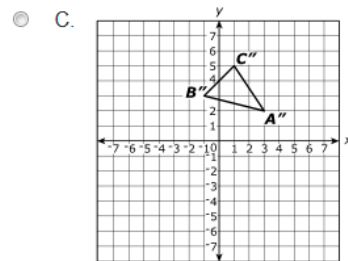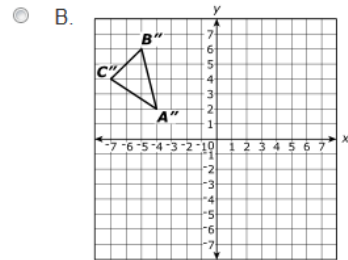5. Triangle *ABC* is graphed in the *xy*-coordinate plane, as shown.

Part A

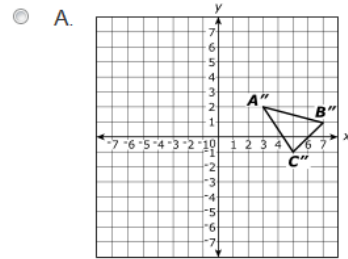Triangle *ABC* is reflected across the *x*-axis to form triangle $A'B'C'$. What are the coordinates of C' after the reflection?

- A. $(-6, 4)$
- B. $(3, -2)$
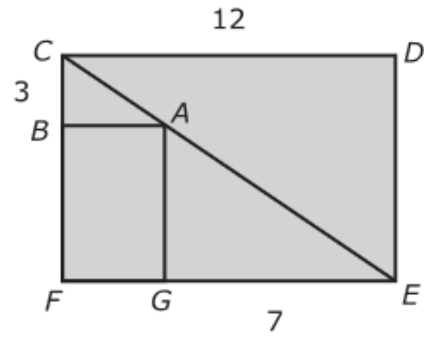- C. $(4, -6)$
- D. $(6, -4)$

Part B

Triangle *ABC* in the *xy*-coordinate plane will be rotated $90°$ counterclockwise about point *A* to form triangle $A''B''C''$. Which graph represents $A''B''C''$?

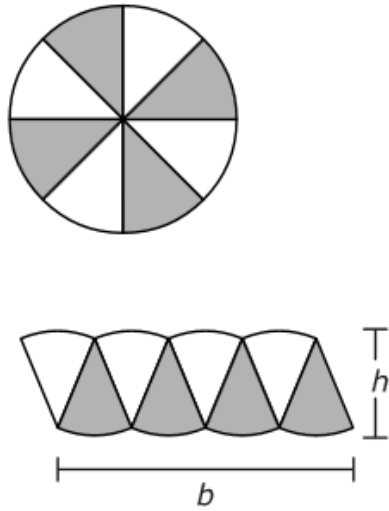- A.

- B.

- C.

- D.

6. In the diagram, quadrilaterals $FBAG$ and $CDEF$ are rectangles.

How long is $\overline{DE}$ rounded to the nearest tenth?

Enter your answer in the box.

7.

The figure illustrates an informal argument for the formula for the area of a circle. The circle is divided into congruent sectors, and the sectors are rearranged to form a shape that resembles a parallelogram, as shown. As the number of sectors increases, the rearranged shape more closely resembles a parallelogram with area A, given by the formula $A = bh$, where b is the base and h is the height of the parallelogram.

Select the correct value for b and h to develop the area of a circle in terms of r, the radius of the circle.

$b =$ [Choose... ▼] , $h =$ [Choose... ▼]

b	h
π	π
r	r
πr	$r \times r$
2π	$2r$
$2\pi r$	2π

8. **Part A**

A circle in the *xy*-coordinate plane has the equation $x^2 + y^2 + 6y - 4 = 0$. If the equation of the circle is written in the form $x^2 + (y + k)^2 = c$, where k and c are constants, what is the value of k?

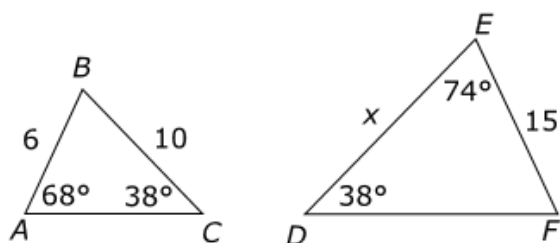Enter your answer in the box.

[ ]

Part B

What is the radius of the circle?

- ⦿ A. 2

- ⦿ B. 4

- ⦿ C. $\sqrt{13}$

- ⦿ D. 13
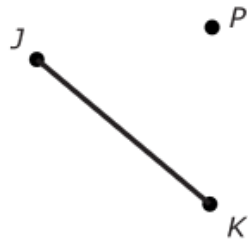
9. Given the two triangles shown, find the value of x.

Select from the drop-down menu to correctly complete the sentence.

The value of x is [Choose.. ▼] .

4
11
12
19
20
25

10. The figure shows line segment JK and a point P that is not collinear with points J and K.

Suppose that line segment $J'K'$ is the image of line segment JK after a dilation with scale factor 0.5 that is centered at point P. Which statement **best** describes the position of line segment $J'K'$?

○ A. Line segment $J'K'$ is parallel to line segment JK.

○ B. Line segment $J'K'$ is perpendicular to line segment JK.

○ C. Line segment $J'K'$ intersects line segment JK at one point, but it is not perpendicular to line segment JK.

○ D. Line segment $J'K'$ lies on the same line as line segment JK.

11. Each of the two-dimensional figures shown will be rotated $360°$ about the respective line, creating a three-dimensional figure.

Drag the appropriate two-dimensional figure to identify the correct representation of the resulting three-dimensional figure.
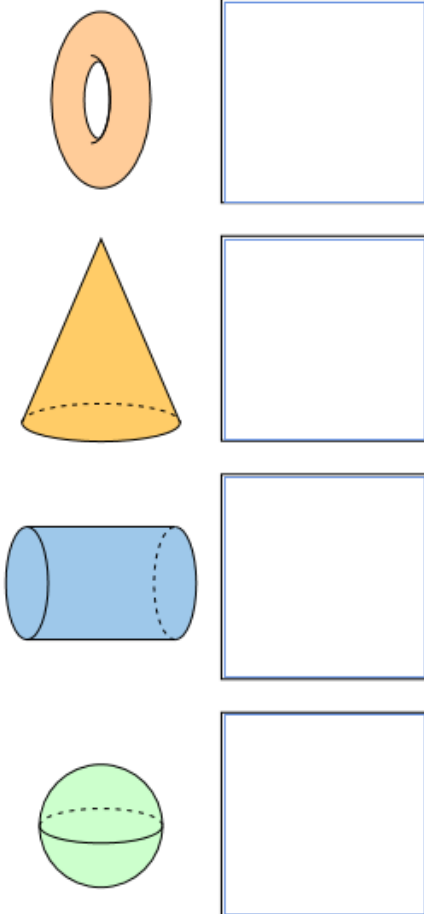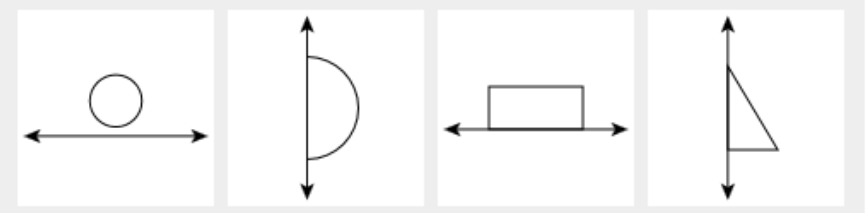
12. The diagram shows $\overline{MN}$ graphed on a coordinate plane.
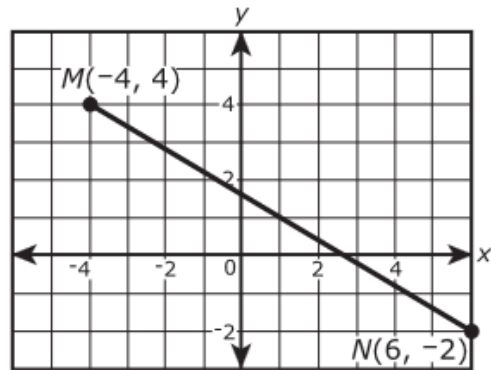
Point P lies on $\overline{MN}$ and is $\frac{3}{4}$ of the way from M to N. What are the coordinates of point P?

Enter your answer in the space provided. Enter **only** your answer.

$(\square , \square)$

13. **Part A**

The number of people who live in a unit of area is called the *population density* of the area. It is usually given as people "per square mile" or "per square kilometer."

A map of the Orchard Hill Neighborhood is shown. The population of the Orchard Hill Neighborhood is 360 people. The length of each block is the same and the length of 20 blocks is 1 mile.

The Orchard Hill Neighborhood

Elm Street

Maple Street

Oak Street

Walnut Street

1st Avenue   2nd Avenue   3rd Avenue   4th Avenue   5th Avenue

total population = 360

length of 20 blocks = 1 mile
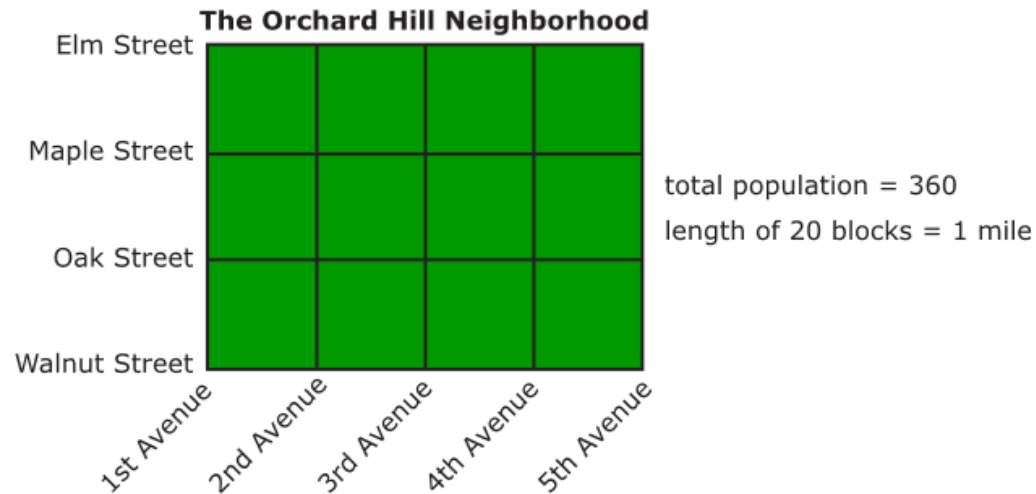
What is the area in square miles of Orchard Hill?

A. 0.03 square mile

B. 0.15 square mile

C. 0.35 square mile

D. 0.60 square mile

Part B

What is the population density of the Orchard Hill Neighborhood, given as a number of people per square mile?

Enter your answer in the box.

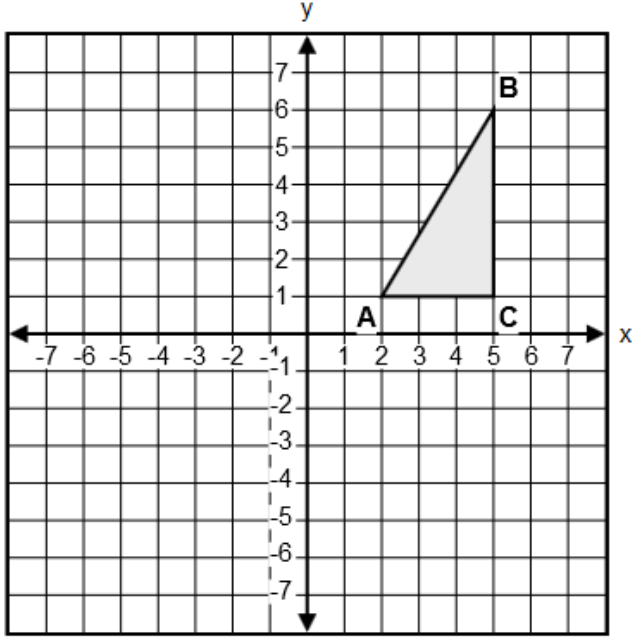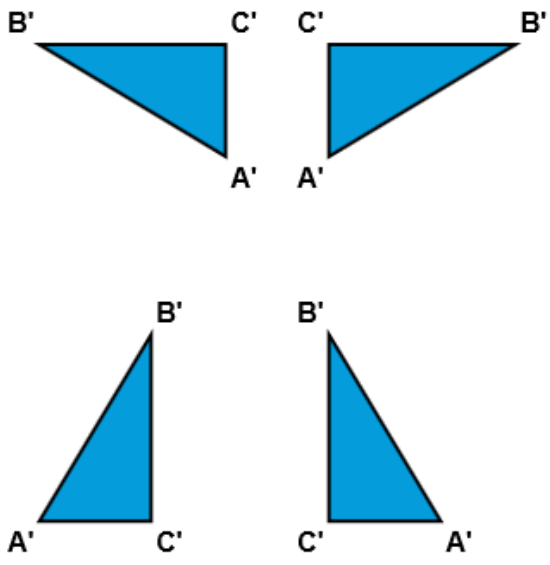14. The right triangle in the coordinate plane is rotated $270°$ clockwise about the point $(2, 1)$ and then reflected across the y-axis to form triangle $A'B'C'$.

Drag and drop the appropriate orientation for triangle $A'B'C'$ into the correct position on the coordinate plane.

15. The given cylindrical container is used to fill the rectangular prism fish tank with water.

8 inches
6 inches
24 inches
24 inches
12 inches
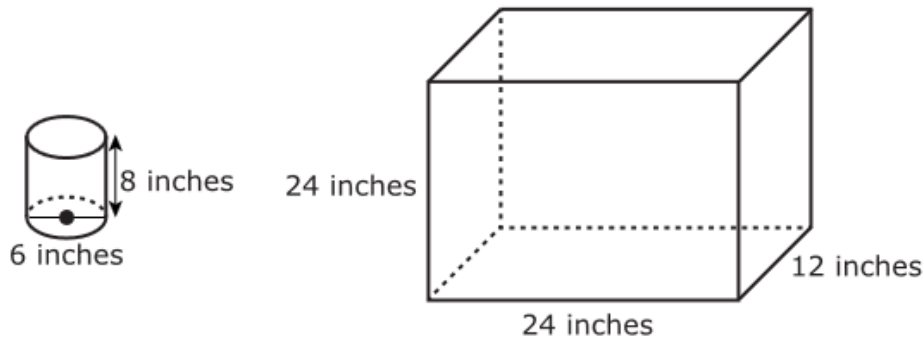
What is the **least** number of full cylindrical containers needed to completely fill the fish tank?

Enter your answer in the box.

⬚ containers

16. Triangle APQ is the image of $\triangle ABC$ under a dilation centered at vertex A with scale factor $\frac{1}{2}$. Triangle RBT is the image of $\triangle ABC$ under a dilation centered at vertex B with scale factor $\frac{3}{4}$. Which statement about $\triangle ABC$, $\triangle APQ$, and $\triangle RBT$ is correct?

- A. All three triangles are similar.

- B. None of the triangles are similar.

- C. Triangles APQ and RBT are not similar because they were dilated using different scale factors.

- D. Triangles APQ and RBT are not similar because they were dilated with different centers of dilation.

17.
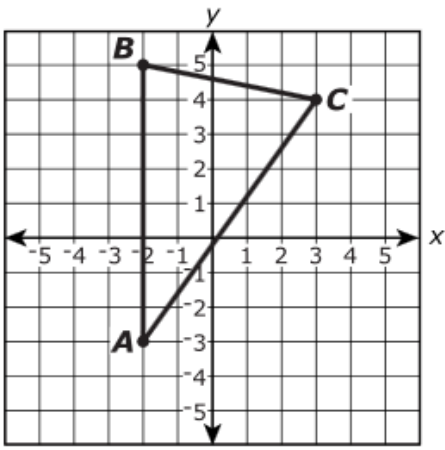
Triangle ABC is shown in the xy-coordinate plane. The triangle will be translated 2 units down and 3 units right to create triangle $A'B'C'$. Indicate whether each of the listed parts of the image will or will not be the same as the corresponding part in the preimage (triangle ABC) by selecting the appropriate box in the table.

	Will be the Same	Will Not be the Same
The coordinates of A'	☐	☐
The coordinates of C'	☐	☐
The perimeter of $\triangle A'B'C'$	☐	☐
The area of $\triangle A'B'C'$	☐	☐
The measure of $\angle B'$	☐	☐
The slope of $\overline{A'C'}$	☐	☐

18. In the figure, $p \parallel s$. Transversals t and w intersect at point L.
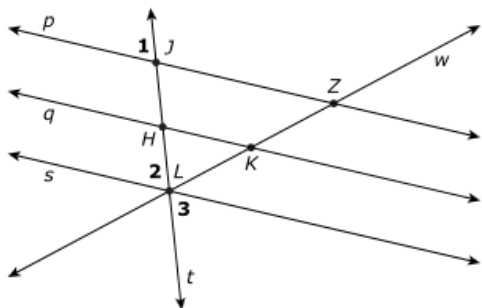
Part A

Statement	Reason
1) $p \parallel s$	Given
2) $\angle 1 \cong \angle 2$	Corresponding angles along parallel lines are congruent.
3) $\angle 2 \cong \angle 3$	?
4) $\angle 1 \cong \angle 3$	Congruence of angles is transitive.

What is the missing reason in step 3?

- A. Alternate interior angles along parallel lines are congruent.

- B. Alternate exterior angles along parallel lines are congruent.

- C. Corresponding angles along parallel lines are congruent.

- D. Vertical angles are congruent.

Part B

Consider the proof of $p \parallel q$ given that $\triangle LHK \sim \triangle LJZ$.

If $\triangle LHK \sim \triangle LJZ$, then $\angle LHK \cong \angle LJZ$ because corresponding angles in similar triangles are congruent.

Which statement concludes the proof?

- A. If $\angle LHK \cong \angle LJZ$, then $p \parallel q$ because when base angles are congruent, the lines are parallel.

- B. If $\angle LHK \cong \angle LJZ$, then $p \parallel q$ because when corresponding angles are congruent, the lines are parallel.

- C. If $\angle LHK \cong \angle LKH$, then $p \parallel q$ because when alternate exterior angles are congruent, the lines are parallel.

- D. If $\angle JLZ \cong \angle HLK$, then $p \parallel q$ because when corresponding angles are congruent, the lines are parallel.

19. Points X and Z are on a number line, and point Y partitions $\overline{XZ}$ into two parts so that the ratio of the length of $\overline{XY}$ to the length of $\overline{YZ}$ is 5:7. The coordinate of X is 1.3, and the coordinate of Y is 3.8. What is the coordinate of Z?

Enter your answer in the box.

20. In circle O, points A, B, C, and D lie on the circle; $\overparen{AD}$ is congruent to $\overparen{BC}$; and the measure of $\overparen{AB}$ is twice the measure of $\overparen{BC}$.
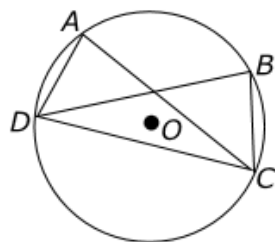
Part A

Select from the drop-down menus to correctly complete the statement.

The measure of $\angle ACD$ is [Choose... ▾] the measure of $\angle ADC$.

- a third
- half
- equal to
- twice
- three times

Part B

Select from the drop-down menus to correctly complete the statement.

The measure of $\angle ADC$ is [Choose... ▾] the measure of $\angle BCD$.

- a third
- half
- equal to
- twice
- three times

21.

$P \bullet\!\!\rule[0.5ex]{7em}{0.4pt}\!\!\bullet Q$

Jericho is making several constructions based on the segment shown.

Part A

$\times$

$P \bullet\!\!\rule[0.5ex]{7em}{0.4pt}\!\!\bullet Q$

$\times$

For his first construction, Jericho made the markings shown with a compass open to a length less than the length of segment $\overline{PQ}$. Jericho's markings are useful for the construction of which of the figures listed?

Select **all** that apply.

☐ A. a $60°$ angle

☐ B. a bisector of $\overline{PQ}$

☐ C. a line perpendicular to $\overline{PQ}$

☐ D. a rhombus with $\overline{PQ}$ as one diagonal

☐ E. an equilateral triangle with side $\overline{PQ}$

Part B
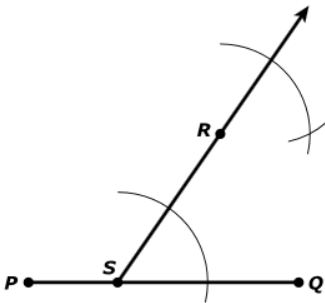
The first steps of Jericho's second construction are shown. After drawing arcs from point S and point R, he adjusted the compass length using the intersection of the arc from point S with $\overline{PQ}$ and $\overrightarrow{SR}$. Which figure is he constructing?

○ A. the bisector of $\overline{PQ}$ through point R

○ B. an angle congruent to $\angle RPQ$ with vertex R

○ C. a line through point R that is parallel to $\overline{PQ}$

○ D. a circle containing points P, Q, and R

22. Hank is putting jelly candies into two containers. One container is a cylindrical jar with a height of 33.3 centimeters and a diameter of 8 centimeters. The other container is spherical. Hank determines that the candies are cylindrical in shape and that each candy has a height of 2 centimeters and a diameter of 1.5 centimeters. He also determines that air will take up 20% of the volume of the containers. The rest of the space will be taken up by the candies.

Part A

After Hank fills the cylindrical jar with candies, what will be the volume, in cubic centimeters, of the air in the cylindrical jar? Round your answer to the nearest whole cubic centimeter.

Enter your answer in the box.

[ ]

Part B

What is the maximum number of candies that will fit in the cylindrical jar?

Enter your answer in the box.

[ ]

Part C

The spherical container can hold a maximum of 260 candies. Approximate the length of the radius, in centimeters, of the spherical container. Round your answer to the nearest tenth.

Enter your answer in the box.

[ ]

Part D

Hank is filling the cylindrical container using bags of candy that have a volume of 150 cubic centimeters. Air takes up 10% of the volume of each bag, and the rest of the volume is taken up by candy. How many bags of candy are needed to fill the cylindrical container with 260 candies?
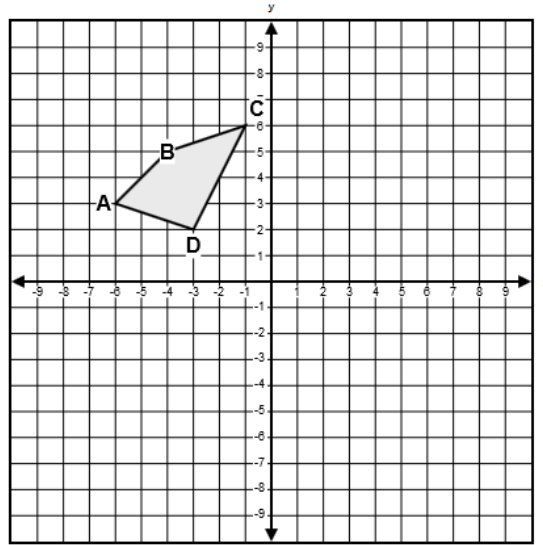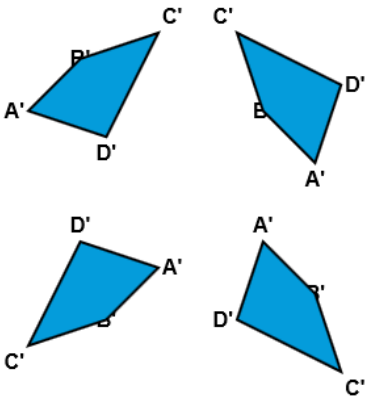
Enter your answer in the box.

[ ]

23. Quadrilateral *ABCD* is shown graphed in the *xy*-coordinate plane.

Part A

Quadrilateral *ABCD* will be translated according to the rule $(x, y) \rightarrow (x + 3, y - 4)$ to form $A'B'C'D'$.

Select the correct orientation of $A'B'C'D'$ and place it correctly in the plane.

Part B

Quadrilateral *ABCD* maps onto $A''B''C''D''$. It will undergo a different transformation that will map $A(-6, 3)$ to A'', $B(-4, 5)$ to B'', $C(-1, 6)$ to C'', and $D(-3, 2)$ to D''. The transformation will consist of a reflection over the *y*-axis followed by a translation. Point D'' is shown plotted in the plane after the transformation.

Plot the point A'' in the plane.
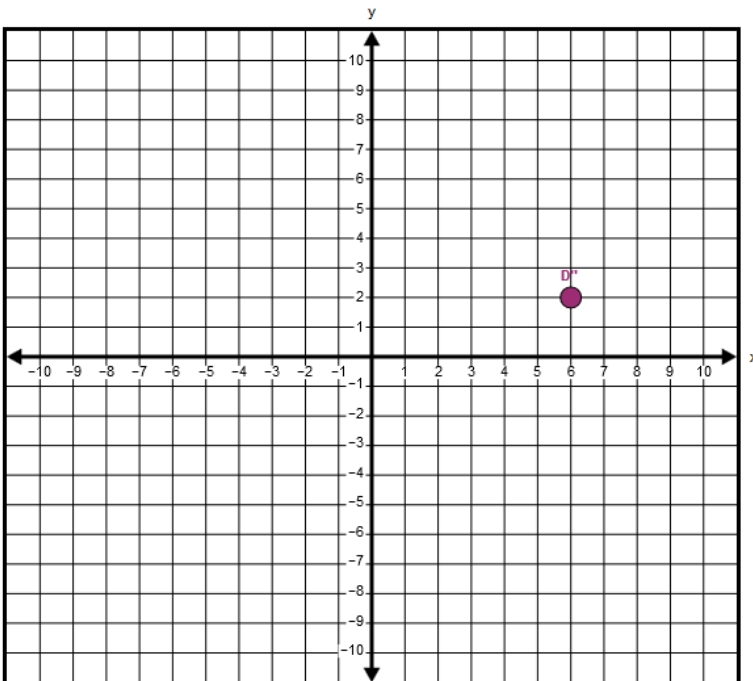
24. A landscaper is designing a display of flowers for an area in a public park. The flower seeds will be planted at points that lie on a circle that has a diameter of 8 feet. The point where any seed is planted must be at least 2 feet away from the seeds on either side of it.

Part A

What is the maximum number of flower seeds that can be planted using the design?

Enter your answer in the box.

[   ] seeds

Part B

After planting the flower seeds, the landscaper has 20 seeds left over. The landscaper wants to plant all of the remaining seeds in another circle so that the seeds are 2 feet apart. To the nearest tenth of a foot, what is the diameter of the smallest circle that the landscaper can use to plant all of the remaining seeds?

Enter your answer in the box.

[   ] feet

25. Using a compass and a straightedge, a student constructed a triangle in which $\overline{XY}$ is one of the sides.

X ———————————————————————— Y
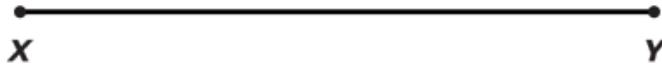
The compass is opened to a set length and two intersecting arcs are drawn above $\overline{XY}$ using X and Y as the centers. The intersection of the two arcs is labeled as point Z.

Part A

What could be the set length of the compass so that $\triangle XYZ$ is isosceles but **not** equilateral?

Select **all** that apply.

☐ A. less than $\frac{1}{2} XY$

☐ B. equal to $\frac{1}{2} XY$

☐ C. between $\frac{1}{2} XY$ and XY

☐ D. equal to XY

☐ E. greater than XY

Part B

Select the correct phrase to complete the sentence.

If the opening of the compass is [Choose... ▼] , then $\triangle XYZ$ will be equilateral.

Less than 1/2 (XY)
equal to 1/2 (XY)
between 1/2 (XY) and (XY)
equal to (XY)
greater than (XY)

26. Which geometric figures have a measurable quantity?

Select **each** correct answer.
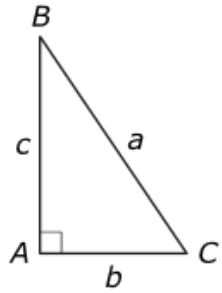
☐ A. line

☐ B. angle

☐ C. point

☐ D. line segment

☐ E. ray

27. A computer monitor is 20 inches wide. The aspect ratio, which is the ratio of the width of the screen to the height of the screen, is 16:9. What is the length of the diagonal of the screen, to the nearest whole inch?

Enter your answer in the box.

28. The figure shows right $\triangle ABC$.

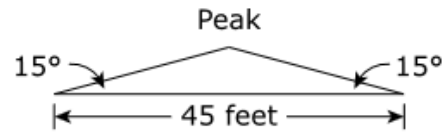Which of the listed values are equal to the sine of B?

Select **all** that apply.

☐ A. $\dfrac{b}{c}$

☐ B. $\dfrac{c}{a}$

☐ C. $\dfrac{b}{a}$

☐ D. the cosine of B

☐ E. the cosine of C

☐ F. the cosine of $(90° - B)$

☐ G. the sine of $(90° - C)$

29. A carpenter is constructing a triangular roof for a storage shed as shown in the figure.

Peak

15°   15°

← 45 feet →

Part A

How high will the peak of the roof rise above the top of the shed?

Give your answer to the nearest foot.

Enter your answer in the box.
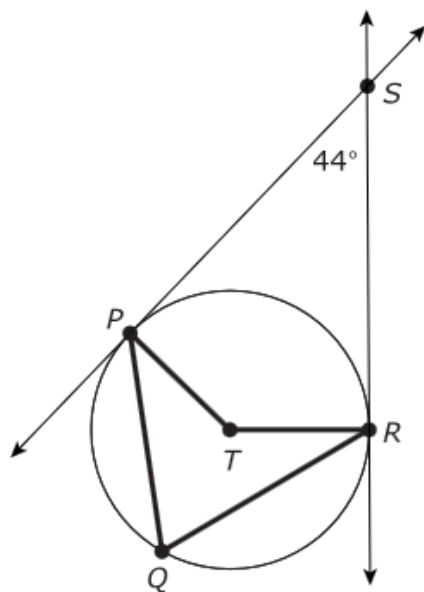
[   ] feet

Part B

After the roof is constructed, it will be covered with an asphalt roofing material. The carpenter needs to calculate the combined length of the two sloping sides. What will be the total length needed of the roof covering?

Give your answer to the nearest foot.

Enter your answer in the box.

[   ] feet

30. Circle T is shown. Line PS and line RS are tangent to circle T.

Part A

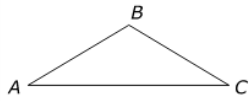What is the measure, in degrees, of $\angle PTR$?

Enter your answer in the box.

Part B

What is the measure, in degrees, of $\angle PQR$?

Enter your answer in the box.

31.

Given: In △ABC shown, $\overline{BA} \cong \overline{BC}$.

Prove: ∠A ≅ ∠C

Statement	Reason
1)	1) Given
2)	2)
3)	3) Definition of midpoint
4)	4)
5)	5)
6)	6)

Part A

Select from the drop-down menus to correctly complete step 2 of the proof.

Choose... ▼ because Choose... ▼ .

let D be the midpoint of line segment AB
let D be the midpoint of line segment AC
let D be the midpoint of line segment BC

every line has exactly one midpoint
every segment has exactly one midpoint
every triangle has exactly one midpoint

Part B

Select from the drop-down menus to correctly complete step 4 of the proof.

Choose... ▼ because of the Choose... ▼ .

angle ADB is congruent to angle CDB
triangle ADB is congruent to triangle CDB
line segment BD is congruent to line segment BD

reflexive property of congruence
definition of perpendicular bisector
Side Angle Side congruence postulate

Part C

Select from the drop-down menus to correctly complete step 5 of the proof.

Choose... ▼ because of the

triangle ABD is similar to triangle CBD
triangle ABD is congruent to triangle CBD
angle ABD is congruent to angle CBD

Angle Angle similarity postulate
Side Side Side congruence postulate
Side Angle Side congruence postulate

.

Part D

What is the correct reason for the statement in step 6?

○ A. the transitive property of congruence

○ B. base angles of isosceles triangles are congruent

○ C. corresponding parts of congruent triangles are congruent

○ D. vertical angles are congruent

32. The figure shows a circle with center P and inscribed isosceles $\triangle ABC$.
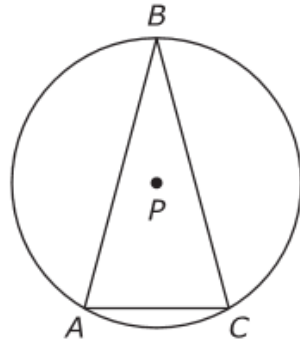
If $\overline{AC}$ has the same length as the radius of the circle, what is the measure of $\angle ABC$?

Enter your answer in the box.

[   ] degrees

33. Triangle JKL will undergo a transformation to create triangle $J'K'L'$ in the xy-coordinate plane. Which transformations will result in $\triangle JKL \cong \triangle J'K'L'$?

Select **all** that apply.

☐ A. $(x, y) \rightarrow (-x, -y)$

☐ B. $(x, y) \rightarrow (-x, y)$

☐ C. $(x, y) \rightarrow (x, y - 5)$

☐ D. $(x, y) \rightarrow (x + 3, y - 5)$

☐ E. $(x, y) \rightarrow (2x, 3y)$

☐ F. $(x, y) \rightarrow (-x, y + 3)$

34. An incomplete proof of the theorem that the sum of the interior angles of a triangle is $180°$ is shown.
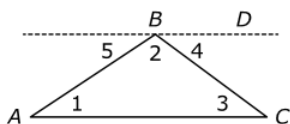
Given: $\triangle ABC$

Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180°$

Statement	Reason
1) Draw line BD parallel to line AC	1)
2)	2)
3) $m\angle 2 + m\angle 4 = m\angle ABD$; $m\angle 5 + m\angle ABD = 180°$	3) Angle addition postulate
4) $m\angle 5 + m\angle 2 + m\angle 4 = 180°$	4) Substitution property of equality
5) $m\angle 1 + m\angle 2 + m\angle 3 = 180°$	5)

Part A

What is the appropriate reason for the statement in step 1 ?

○ A. Through any two points, there is exactly one line.

○ B. Through a point not on a line, there is exactly one line parallel to the given line.

○ C. If two lines cut by a transversal form congruent corresponding angles, then the lines are parallel.

○ D. If two lines cut by a transversal form congruent alternate interior angles, then the lines are parallel.

Part B

Which pairs of angle congruences or equalities should be used for the statement in step 2 ?

Indicate **all** such pairs.

☐ A. $\angle 1 \cong \angle 2$ or $m\angle 1 = m\angle 2$

☐ B. $\angle 1 \cong \angle 3$ or $m\angle 1 = m\angle 3$

☐ C. $\angle 1 \cong \angle 4$ or $m\angle 1 = m\angle 4$

☐ D. $\angle 1 \cong \angle 5$ or $m\angle 1 = m\angle 5$

☐ E. $\angle 2 \cong \angle 3$ or $m\angle 2 = m\angle 3$

☐ F. $\angle 2 \cong \angle 4$ or $m\angle 2 = m\angle 4$

☐ G. $\angle 2 \cong \angle 5$ or $m\angle 2 = m\angle 5$

☐ H. $\angle 3 \cong \angle 4$ or $m\angle 3 = m\angle 4$

Part C

Select from the drop-down menu to correctly complete the sentence.

The reason for the statement in step 2 is that [Choose... ▼] .

If two parallel lines are cut by a transversal, then alternate interior angles are congruent
If two parallel lines are cut by a transversal, then corresponding angles are congruent
It two lines cut by a transversal form congruent corresponding angles, then the lines are parallel
It two lines cut by a transversal form congruent alternate interior angles, then the lines are parallel

Part D

Select from the drop-down menu to correctly complete the sentence.

The appropriate reason for the statement in step 5 is the [Choose... ▼] .

Reflexive property of equality
symmetric property of equality
transitive property of equality
substitution property of equality