Math
Spring Operational 2015

Integrated Mathematics III
End of Year Released Items

1. What are the zeros of the polynomial $x(x^2 + 4x - 12)$?

   Indicate **all** zeros.

   - [ ] A. -12
   - [ ] B. -6
   - [ ] C. -3
   - [ ] D. -2
   - [ ] E. 0
   - [ ] F. 2
   - [ ] G. 6
   - [ ] H. 12

2. The function f is defined as $f(x) = x(x^2 - 4) - 3x(x - 2)$.

Part A

An equivalent form of f is given as $f(x) = x(x - 2)(x - a)$, where a is a constant. What is the value of a?

Enter your answer in the box.

$a =$ ☐

Part B

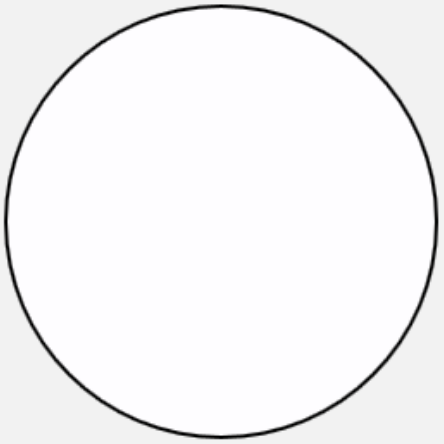
Which values are the zeros of the function f?

Select **all** that apply.

- ☐ A. -3
- ☐ B. -2
- ☐ C. -1
- ☐ D. 0
- ☐ E. 1
- ☐ F. 2
- ☐ G. 3

3. The circle has a radius of 12 units. Shade an area of 24π square units.

Divide the circle into the correct number of sections by selecting the "More" button. If you divide the circle into too many sections, use the "Fewer" button. Then, select the number of sections to represent the answer.



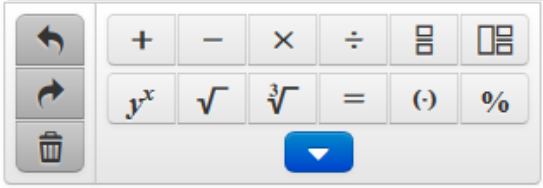
Fewer More Reset

4. The equation $x^2 - 10x + 17 = -y^2 - 2y$ describes a circle in the coordinate plane.

Find the radius of the circle and the coordinates of its center.

Enter your answers in the spaces provided. Enter **only** your answers.

$\textbf{radius} = \square$ **units** $\quad \textbf{center} : (\square, \square)$



5. **Part A**

A circle in the *xy*-coordinate plane has the equation $x^2 + y^2 + 6y - 4 = 0$. If the equation of the circle is written in the form $x^2 + (y + k)^2 = c$, where *k* and *c* are constants, what is the value of *k*?

Enter your answer in the box.

Part B

What is the radius of the circle?

- A. 2
- B. 4
- C. $\sqrt{13}$
- D. 13

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6. Let $f(x) = 2x^2 - x + 1$ and let $g(x) = x - 4$. Which statements are true?

Select **all** that apply.

- [ ] A. One root of $f(x)$ is -4.
- [ ] B. One root of $f(x)$ is 29.
- [ ] C. If $f(x)$ is divided by $g(x)$, the remainder is 29.
- [ ] D. $g(x)$ is a factor of $f(x)$.
- [ ] E. $g(x)$ is not a factor of $f(x)$.

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7. Solve $\sqrt{a} = a - 6$. What is the extraneous solution?

Enter your answer in the box.

[ ]

8. Consider the equation $p^2 - 5p - 6 - x(p-6)^2 = 0$, where p is a real constant.

Part A

If $p = 6$, then the equation has

- A. no real solutions.
- B. exactly one real solution.
- C. exactly two real solutions.
- D. infinitely many real solutions.

Part B

If $p \neq 6$, then $x =$

- A. $\frac{p-2}{p-6}$
- B. $\frac{p-1}{p-6}$
- C. $\frac{p+1}{p-6}$
- D. $\frac{p+2}{p-6}$

9. A solution for each equation is given.

Drag and drop the letter of each equation into the appropriate box for its solution.

A B

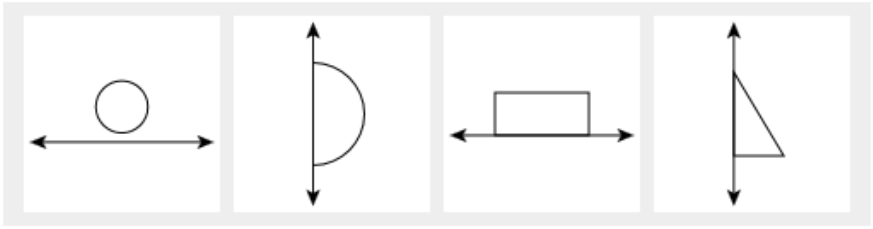
A $-4x + 20 - a = 4(-x + 5) + a$

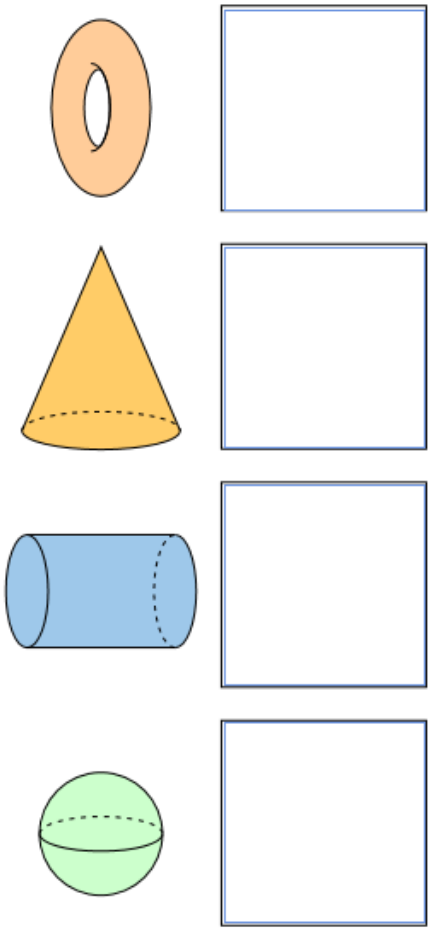
B $4x - 20 + 2a = 4(x - 5) + a + 1$

$a = 1$ $a = 0$ $a = -1$

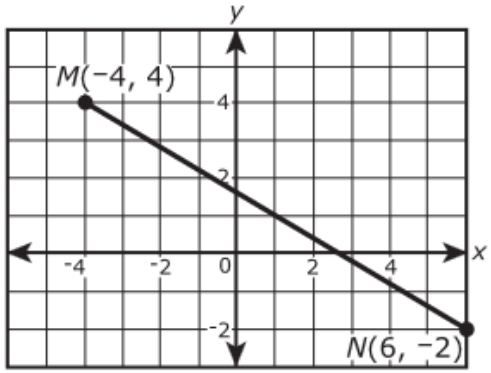
10. Each of the two-dimensional figures shown will be rotated 360° about the respective line, creating a three-dimensional figure.

Drag the appropriate two-dimensional figure to identify the correct representation of the resulting three-dimensional figure.





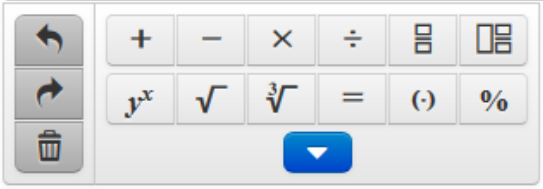
11. The diagram shows $\overline{MN}$ graphed on a coordinate plane.



Point P lies on $\overline{MN}$ and is $\frac{3}{4}$ of the way from M to N. What are the coordinates of point P?

Enter your answer in the space provided. Enter **only** your answer.

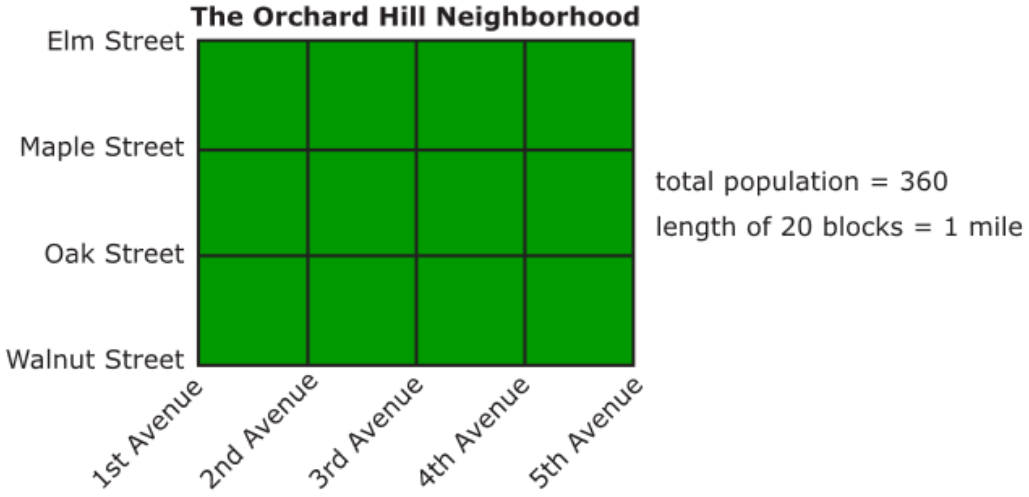
$(\square, \square)$



12. **Part A**

The number of people who live in a unit of area is called the *population density* of the area. It is usually given as people "per square mile" or "per square kilometer."

A map of the Orchard Hill Neighborhood is shown. The population of the Orchard Hill Neighborhood is 360 people. The length of each block is the same and the length of 20 blocks is 1 mile.



What is the area in square miles of Orchard Hill?

- A. 0.03 square mile
- B. 0.15 square mile
- C. 0.35 square mile
- D. 0.60 square mile

Part B

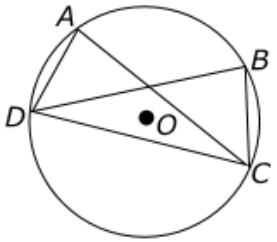
What is the population density of the Orchard Hill Neighborhood, given as a number of people per square mile?

Enter your answer in the box.

13. Points X and Z are on a number line, and point Y partitions $\overline{XZ}$ into two parts so that the ratio of the length of $\overline{XY}$ to the length of $\overline{YZ}$ is 5:7. The coordinate of X is 1.3, and the coordinate of Y is 3.8. What is the coordinate of Z?

Enter your answer in the box.

14. In circle O, points A, B, C, and D lie on the circle; $\overset{\frown}{AD}$ is congruent to $\overset{\frown}{BC}$; and the measure of $\overset{\frown}{AB}$ is twice the measure of $\overset{\frown}{BC}$.



Part A

Select from the drop-down menus to correctly complete the statement.

The measure of $\angle ACD$ is [Choose...] the measure of $\angle ADC$.

- a third
- half
- equal to
- twice
- three times

Part B

Select from the drop-down menus to correctly complete the statement.

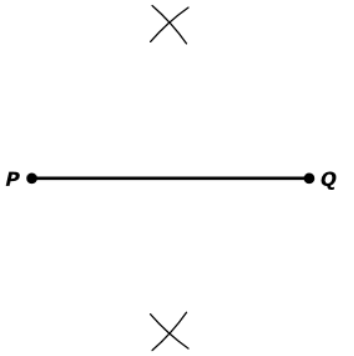
The measure of $\angle ADC$ is [Choose...] the measure of $\angle BCD$.

- a third
- half
- equal to
- twice
- three times

15.

Jericho is making several constructions based on the segment shown.

Part A

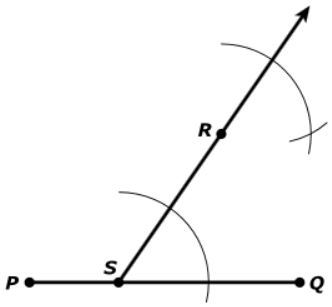


For his first construction, Jericho made the markings shown with a compass open to a length less than the length of segment $\overline{PQ}$. Jericho's markings are useful for the construction of which of the figures listed?

Select **all** that apply.

- [ ] A. a 60° angle
- [ ] B. a bisector of $\overline{PQ}$
- [ ] C. a line perpendicular to $\overline{PQ}$
- [ ] D. a rhombus with $\overline{PQ}$ as one diagonal
- [ ] E. an equilateral triangle with side $\overline{PQ}$

Part B



The first steps of Jericho's second construction are shown. After drawing arcs from point S and point R, he adjusted the compass length using the intersection of the arc from point S with $\overline{PQ}$ and $\overrightarrow{SR}$. Which figure is he constructing?

- ○ A. the bisector of $\overline{PQ}$ through point R
- ○ B. an angle congruent to $\angle RPQ$ with vertex R
- ○ C. a line through point R that is parallel to $\overline{PQ}$
- ○ D. a circle containing points P, Q, and R

16. Hank is putting jelly candies into two containers. One container is a cylindrical jar with a height of 33.3 centimeters and a diameter of 8 centimeters. The other container is spherical. Hank determines that the candies are cylindrical in shape and that each candy has a height of 2 centimeters and a diameter of 1.5 centimeters. He also determines that air will take up 20% of the volume of the containers. The rest of the space will be taken up by the candies.

Part A

After Hank fills the cylindrical jar with candies, what will be the volume, in cubic centimeters, of the air in the cylindrical jar? Round your answer to the nearest whole cubic centimeter.

Enter your answer in the box.

Part B

What is the maximum number of candies that will fit in the cylindrical jar?

Enter your answer in the box.

Part C

The spherical container can hold a maximum of 260 candies. Approximate the length of the radius, in centimeters, of the spherical container. Round your answer to the nearest tenth.

Enter your answer in the box.

Part D

Hank is filling the cylindrical container using bags of candy that have a volume of 150 cubic centimeters. Air takes up 10% of the volume of each bag, and the rest of the volume is taken up by candy. How many bags of candy are needed to fill the cylindrical container with 260 candies?

Enter your answer in the box.

17. A landscaper is designing a display of flowers for an area in a public park. The flower seeds will be planted at points that lie on a circle that has a diameter of 8 feet. The point where any seed is planted must be at least 2 feet away from the seeds on either side of it.

Part A

What is the maximum number of flower seeds that can be planted using the design?

Enter your answer in the box.

[ ] seeds

Part B

After planting the flower seeds, the landscaper has 20 seeds left over. The landscaper wants to plant all of the remaining seeds in another circle so that the seeds are 2 feet apart. To the nearest tenth of a foot, what is the diameter of the smallest circle that the landscaper can use to plant all of the remaining seeds?

Enter your answer in the box.

[ ] feet

18. Using a compass and a straightedge, a student constructed a triangle in which $\overline{XY}$ is one of the sides.

The compass is opened to a set length and two intersecting arcs are drawn above $\overline{XY}$ using X and Y as the centers. The intersection of the two arcs is labeled as point Z.

Part A

What could be the set length of the compass so that $\triangle XYZ$ is isosceles but **not** equilateral?

Select **all** that apply.

- ☐ A. less than $\frac{1}{2}\,XY$
- ☐ B. equal to $\frac{1}{2}\,XY$
- ☐ C. between $\frac{1}{2}\,XY$ and XY
- ☐ D. equal to XY
- ☐ E. greater than XY

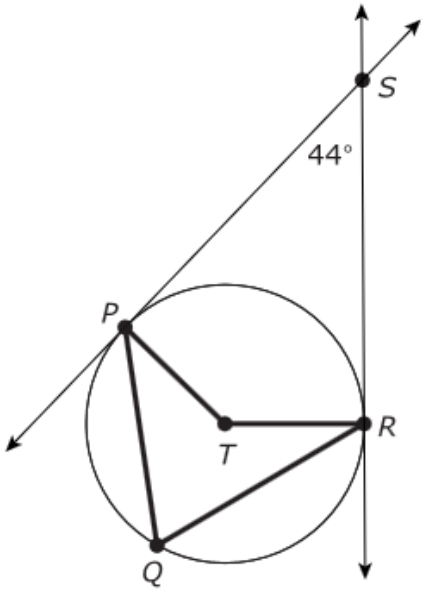
Part B

Select the correct phrase to complete the sentence.

If the opening of the compass is Choose..., then $\triangle XYZ$ will be equilateral.

Less than 1/2 (XY)
equal to 1/2 (XY)
between 1/2 (XY) and (XY)
equal to (XY)
greater than (XY)

19. Circle T is shown. Line PS and line RS are tangent to circle T.



Part A

What is the measure, in degrees, of $\angle PTR$?

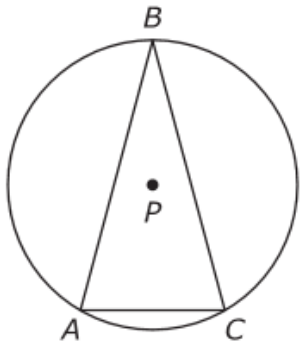
Enter your answer in the box.

Part B

What is the measure, in degrees, of $\angle PQR$?

Enter your answer in the box.

20. The figure shows a circle with center P and inscribed isosceles $\triangle ABC$.

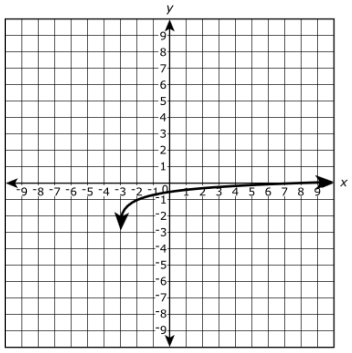


If $\overline{AC}$ has the same length as the radius of the circle, what is the measure of $\angle ABC$?

Enter your answer in the box.

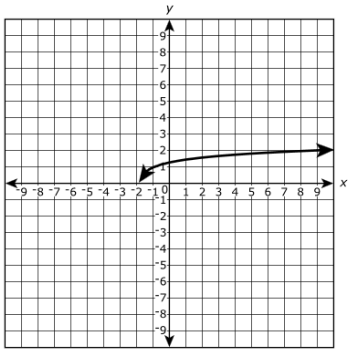
[ ] degrees

21. The given graph represents the function $f(x)$. Let g be defined as $g(x) = f(x-1) + 2$.

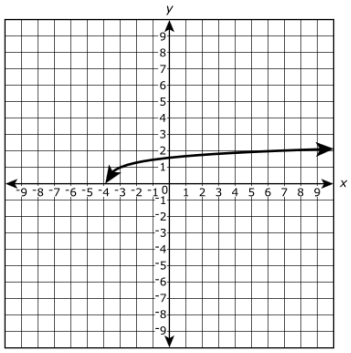


What is the graph of $g(x)$?

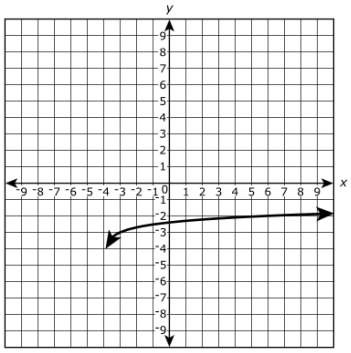
A.



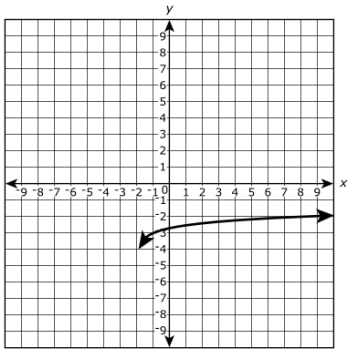
B.



C.



D.



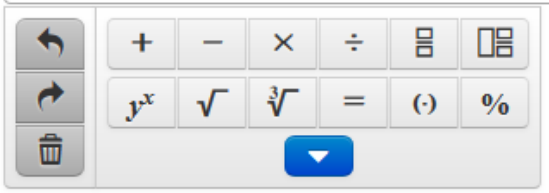
22. Joanna uses the function $p = f(n) = 30n - 450$ to calculate the profit, p, in dollars that she makes from selling n cakes in her store.

- Write a formula for a function to calculate the number of cakes Joanna needs to sell for a given profit.
- Calculate the minimum number of cakes that Joanna must sell to make a profit of at least $500.00.

Enter your answers in the spaces provided. Enter **only** your answers.

Formula: $n = f^{-1}(p) =$ ☐

Minimum number of cakes: ☐



23. There is a unique quadratic function of the form $f(x) = ax^2 + c$ that satisfies each of these conditions:

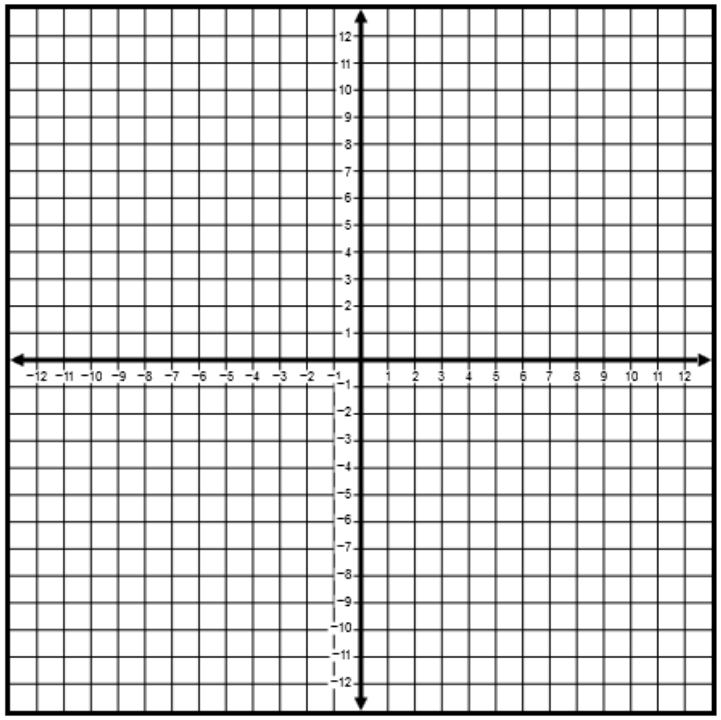
- $f(-2) = f(2) = 0$
- f attains a maximum value of 8

Part A

Create a graph of $f(x)$.

1. Select the quadratic button.
2. Drag the vertex and another point to graph the function.

Quadratic



Part B

Select from the drop-down menus to correctly complete the sentence.

The function f is symmetric about [Choose...] because for all values of x, $f(-x) =$ [Choose..].

Choose...
- the x-axis
- the y-axis
- the origin

Choose..
- -f(-x)
- -f(x)
- f(x)

24. Suppose that θ is a second quadrant angle and that $\cos\theta = -\frac{4}{5}$. What is the value of $\sin\theta$ to the nearest tenth?

Enter your answer in the box.

25. The organizers of a community fair set up a small Ferris wheel for young children. The table shows the heights of one of the cars above the ground for different rotations of the wheel.

Angle of Rotation (radians)	Height above the Ground (feet)
0	1
$\frac{\pi}{2}$	7
π	13
$\frac{3\pi}{2}$	7
2π	1
$\frac{5\pi}{2}$	7
3π	13
$\frac{7\pi}{2}$	7
4π	1

Part A

The function $h(x) = a\ \sin\left(x - \frac{\pi}{2}\right) + b$, where *a* and *b* are constants, models the height of the Ferris wheel car at a rotation of *x* radians.

What are the values of *a* and *b*?

- A. $a = 1;\ b = 12$
- B. $a = 6;\ b = 7$
- C. $a = 7;\ b = 6$
- D. $a = 12;\ b = 1$

Part B

Consider the graph of $y = h(x)$ in the *xy*-coordinate plane. Which statements are true?

Select **all** that apply.

- A. The amplitude of the graph is 12.
- B. The period of the graph is 2π.
- C. The midline of the graph is at $y = 13$.
- D. The graph is increasing for $4\pi < x < 5\pi$.
- E. The graph is decreasing for $\frac{11\pi}{2} < x < \frac{13\pi}{2}$.
- F. The graph has a maximum at $y = 13$.

26. Mr. and Mrs. Ryan build and sell nesting boxes for small birds. They sell each box for $19.95. So far this month, they have built and sold 74 boxes.

Part A

Let x represent the number of additional boxes they expect to build and sell for the month. Let $n(x)$ represent the amount of money they expect to receive from the entire month's sales. Write an equation for $n(x)$.

Enter your equation in the space provided. Enter **only** your equation.

$n(x) = \square$

Part B

Determine the average rate of change, in dollars per nesting box, of $n(x)$ as x varies from 85 to 120.

Enter your answer in the box.

Part C

Mr. and Mrs. Ryan can build no more than 450 boxes in one month. Which statement gives the domain of function n?

A. $0 \leq x < 450$, where x is a whole number

B. $0 \leq x \leq 450$, where x is a whole number

C. $0 < x < 376$ where x is a whole number

D. $0 \leq x \leq 376$, where x is a whole number

Part D

The materials for each nesting box cost a total of $11.75. Write a function $P(s)$ for the profit the Ryans earn when they sell s nesting boxes.

Enter your function in the space provided. Enter **only** your function.

$P(s) = \square$

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27. Melissa has a savings account. She deposited $1,000 into the account the first year. For each year after the first, she plans to deposit an amount that is 2 percent greater than the amount deposited the preceding year. If she makes no other deposits, the total amount of the deposited money in year *n* is the sum S_n of a geometric series of *n* terms.

Part A

The formula for S_n can be expressed as $\frac{1{,}000(1-r^n)}{1-r}$. Use the information given about Melissa's account to determine the value of *r*.

Enter your answer in the box.

Part B

Melissa will have deposited approximately how much by year 30?

- A. $30,000
- B. $35,729
- C. $40,568
- D. $87,453

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28. Which expression is equivalent to $162x^4 - 144x^2 + 32$?

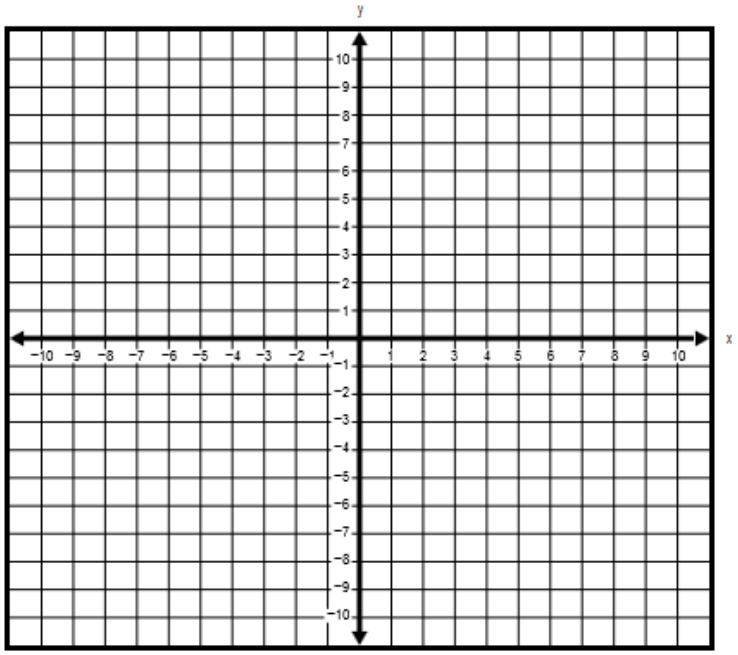
Select **all** that apply.

- A. $2(81x^2 - 72x + 16)$
- B. $2(81x^2 + 4)(81x^2 + 4)$
- C. $2(81x^2 - 4)(81x^2 + 4)$
- D. $2(9x^2 - 4)(9x^2 - 4)$
- E. $2(9x^2 + 4)(9x^2 + 4)$
- F. $2(3x + 2)^2(3x - 2)^2$

29. Create the approximate graph of the quadratic function with x-intercepts at $(-5, 0)$ and $(3, 0)$ and a y-intercept at $(0, -7.5)$.

1. Select a button to choose the graph type.
2. Drag the two points to the correct position.

Quadratic



30. A quiz in math class consisted of 5 true or false questions. Only 4 out of 30 students in Mr. Denario's class got all 5 questions correct. Mr. Denario used a simulation of flipping a coin to represent the results of his class quiz. He used heads to represent a correct answer and tails to represent an incorrect answer. He flipped 5 coins to represent the 5 problems on the quiz and recorded the number of heads in each group of 5 coins. He repeated the simulation 100 times and recorded the results in this table.

Total Number of Heads	Number of Samples (out of 100 samples)
0	2
1	25
2	22
3	34
4	13
5	4

Which conclusion is supported by the data for the class and the simulation?

A. The simulation model is consistent with the class data because in both the observed class results and the simulation model the result of all heads on all 5 problems correct was 4.

B. The simulation model is consistent with the class data because each coin flipped and each question on the test had only two outcomes and an equal chance of getting either outcome.

C. The simulation model is not consistent with the class data because there are 30 students in the class and there were 100 samples in the coin simulation.

D. The simulation model is not consistent with the class data because the observed class results had 13% of the students with all 5 problems correct, and the simulation model only had 4% of the samples with all 5 heads.

VF649980

31. The weight of a bag of Brand A cookies is labeled as 4 ounces on the bag. However, the actual weights of the bags vary by a small amount. According to the packaging specifications, the weights are approximately normally distributed with a mean of 4.10 ounces and a standard deviation of 0.10 ounce.

Part A

Select the number to complete the sentence.

According to the specifications, approximately Choose.. percent of the bags weigh 4.00 ounces or more.

- 25
- 34
- 50
- 84

Part B

During a quality control check on the bag weights, a bag was found that weighed 3.95 ounces. How many standard deviations below the mean was the bag weight?

Enter your answer in the box.

[ ] standard deviations

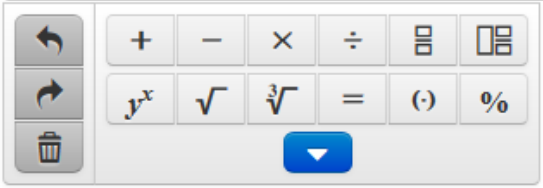
M42696

32. Data were collected on the number of kilowatt-hours (kWh) of electricity customers used each month. The data were fit into a function. The function $f(x) = 650\sin(0.6x - 2.8) + 962$ gives the number of kWh of electricity used during month x. Month 1 corresponds to January.

The electric company charges $0.0715 per kWh up to 500 kWh. Then the company charges $0.1042 per kWh for kWh used above 500. The company also charges $8.00 in fees. How many kWh would a customer use in April, and what would the total cost be?

Enter your answers in the space provided. Enter **only** your answers.

kWh used in April : ☐
Total cost for April : ☐



33. The management of a furniture store chain wanted to determine which color of couch adults in the United States preferred. The management conducted a survey of a random sample of adults in the United States. The number of people who preferred each color is shown in the table.

Color of Couch Preferred

Color	Number of People
black	143
blue	108
brown	210
green	93
red	126
white	204

Part A

What is the population of the study?

- A. the color of couch preferred
- B. all adults in the United States
- C. 884 adults selected at random
- D. the adults surveyed who preferred blue

Part B

What is the estimated proportion of the population that prefers brown couches? Round your answer to the nearest tenth of a percent.

Enter your answer in the box.

[ ] %

34. Let $f(x) = -x + 3$ and $g(x) = 3|x| - 1$. Where do the graphs of $f(x)$ and $g(x)$ intersect?

Enter your answer in the boxes.

(☐ , ☐) (☐ , ☐)

35. **Part A**

Write the expression $j^4 - 16 + (j^2 - 4)^2$ without parenthesis and with all like terms combined.

Enter your answer in the space provided.

Part B

Which expressions are equivalent to the expression $j^4 - 16 + (j^2 - 4)^2$?

Select **all** that apply.

- ☐ A. $(j^2 - 4)(j^2 + 4)$
- ☐ B. $2j^2(j + 2)(j - 2)$
- ☐ C. $(j - 2)(j + 2)(j^2 + 4)$
- ☐ D. $(j^2 + 4)(j^2 - 4)(j^2 - 4)^2$
- ☐ E. $(j^2 - 4)(j^2 + 4 + j^2 - 4)$
- ☐ F. $(j^2 + 4)(j^2 - 4) + (j^2 - 4)^2$

36. A company is starting a scholarship fund for its employees. The company will add to the fund on a yearly basis. Each year, it will increase the amount added to the fund the previous year by 10%. The amount in the scholarship fund after 4 years is $2,552.55.

Part A

To the nearest dollar, how much did the company add to the scholarship fund in the first year?

- A. $182
- B. $550
- C. $742
- D. $2,298

Part B

If the company decided to change the amount added the first year to $1,000 and then increase the amount added by 10% each year, what would be the average amount of whole dollars per year added to the fund over the first 10 years?

Enter your answer in the box.